HW 1

This is due Friday February 2nd.

1. Show that the map $f\colon \mathbb{N}^2 \rightarrow \mathbb{N}$ given by:

$$f(x,y) = x + \frac{(x+y).(x+y+1)}{2}$$

is surjective. (We already proved it was injective, therefore it will be bijective).

2. Is the number $\sqrt{2} + \sqrt{3}$ a rational number?

3. Prove by induction that

$$\sum_{i=1}^{n} k^3 = \left[\frac{n.(n+1)}{2}\right]^2.$$

4. Prove that $n^5 - n$ is divisible by 30, for any integer n.

5. We have seen examples of "twin primes" p, q (their difference is equal to ± 2). Let p and q be two primes: show that pq + 1 is the square of an integer if and only if p and q are twin primes.

6. Draw the hyperbola $\mathcal{H}$: $\left\{(x,\, y) \in \mathbb{R}^2 \big/ x^2 - y^2 = 1\right\}$. Find all the points on $\mathcal{H}$ that have rational coordinates.

Hint: Take the half-lines starting from $(-1, 0)$ and having a rational slope and see where they intersect the hyperbola.