MAT 130 Homework 1
Due Feb. 4, 2005

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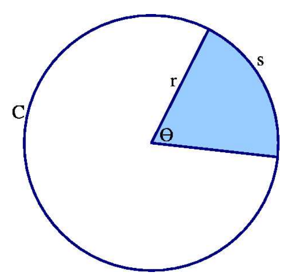
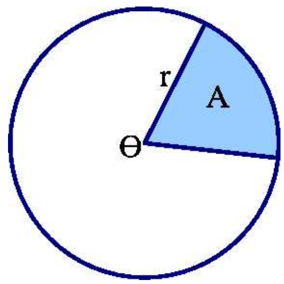
1 RIGHT TRIANGLE RATIOS

1.1 Angles, Degrees, and Arcs

1. Discuss the meaning of an angle of 1 degree.
2. Of the two angles 80.668° and $80^\circ 40' 20''$, whichis larger, or are they equal?
3. find θ exactly, if $s = 12m$ and $C = 108m$.

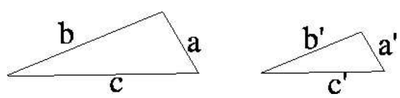


4. If $r = 32.4in$ and $A = 347in^2$, then what is θ to two decimal places? The ratio of the area of a wedge to the area of the whole circle, πr^2, is the same as the ratio of the angle θ to $360°$.



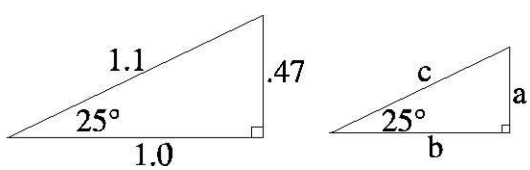
1.2 Similar Triangles

1. Given the two similar triangles, if $b = 3, c = 24$, and $b' = 1$, then what is c'?

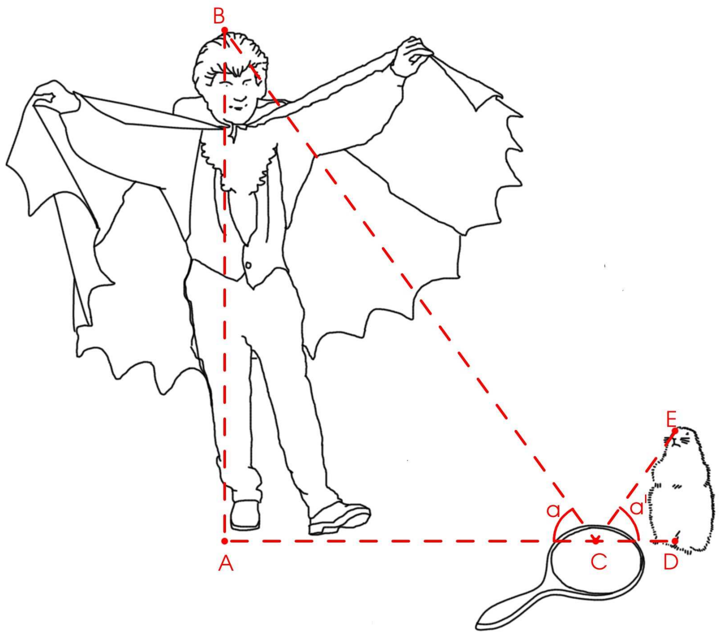


2. If two triangles are similar and a side of one triangle is equal to the corresponding side of the other, are the remaining sides of the two triangles equal? Explain.

3. Given the two similar triangles, if $b = 32cm$, then what are a and c to two significant digits?



4. The prairie is under attack by a giant vampire. This prairie dog has put a mirror on the ground to try to measure the height of the giant vampire. Unfortunately, vampires cast no reflection, but if they did then the tip of the giant vampire's head would reflect in the mirror so that the angles would be the same. Find the height of the vampire given that AC=24 ft, CD=2.1 ft, and DE, the height of the prairie dog, is 5.5 ft.



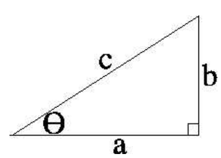
1.3 Trigonometric Ratios and Right Triangles

1. Find $\cos 18.9°$ to three significant digits.

2. If θ is acute and $\tan\theta = 1$, then what is θ to the nearest degree?

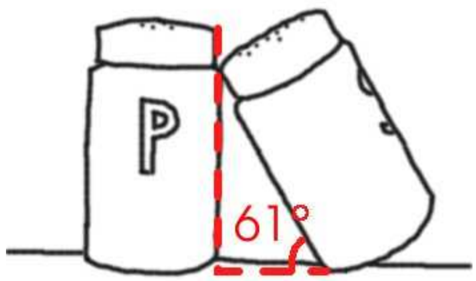
3. If you are give the measures of one side and one acute angle in a right triangle, explain why you can or cannot solve the triangle.

4. If $\theta = 32.4°$ and $a = 42.3m$, the solve the right triangle.

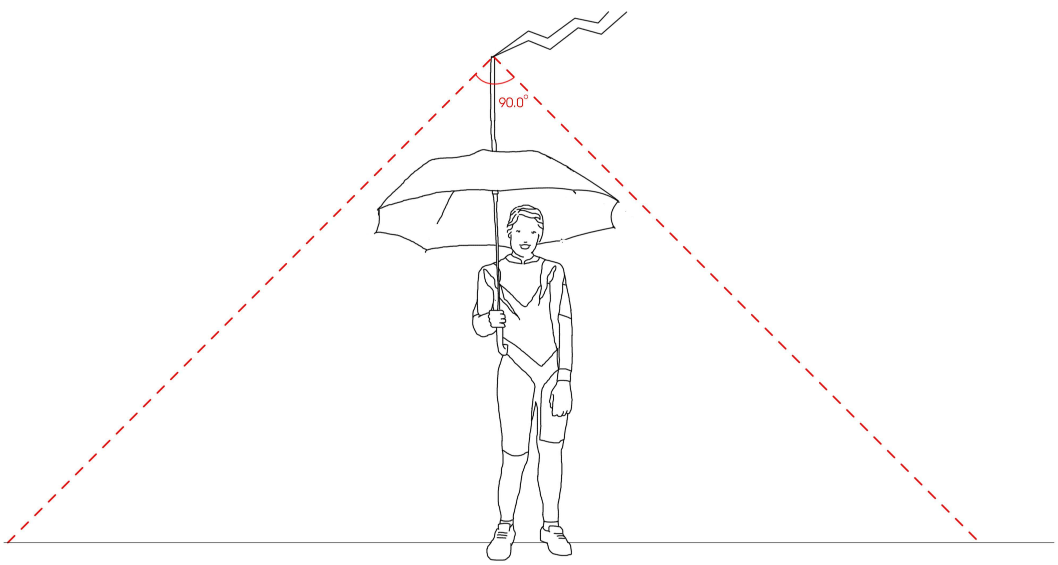


1.4 Right Triangle Applications

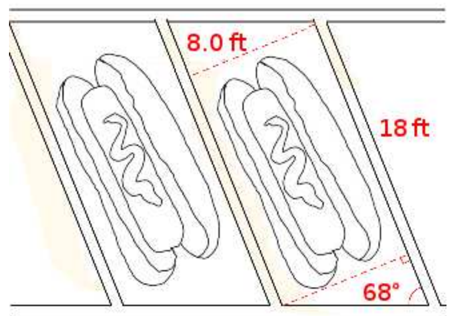
1. A giant's salt shaker 8.0m long is placed against a giant's pepper shaker as indicated in the figure. How high will the top of the giant's salt shaker reach up the giant's pepper shaker?



2. A grounded lightning rod on the head of an giantess' umbrella produces a cone of safety as indicated in the figure. If the top of the rod is 67.0 ft above the ground, what is the diameter of the circle of safety on the ground?



3. To accomodate giant hot dogs of most sizes, a parking space needs to contain an 18 ft by 8.0 ft rectangle as shown in the figure. If a diagonal parking space makes an angle of 68° with the horizontal, how long are the sides of the parallelogram that contains the rectangle?



2 TRIGONOMETRIC FUNCTIONS

2.1 Degrees and Radians

1. Which is smaller: an angle of degree measure 20 or an angle of radian measure 1/2? Explain.
2. Sketch a 45° angle in standard position and find the degree measure of the two nearest angles, one positive, the other negative, which look the same as a 45° angle.
3. Find the radian measure of 9° in exact form and to four significant digits.
4. Find the degree measure of $\pi/60$ radians exactly and to four significant digits.
5. Which quadrant (I,II,III, or IV) does the terminal side of the angle $9\pi/4$ lie in?

2.2 Trigonometric Functions

1. Find the exact value of each of the six trigonometric functions if the terminal side of θ contains the point $(-3, -4)$. How about $(-9, -12)$?
2. If $\sin\theta$ is $3/5$ and θ is a quadrant II angle, what are the values of the other five trigonometric functions? (Draw a reference triangle)

3. Find the exact value of each of the six trigonometric functions for an angle θ that has a terminal side containing $(-1, \sqrt{3})$.

4. In which quadrants must the the terminal side of an angle θ lie in order that $\sin\theta$ be positive?

5. If $\tan\theta$ is -2 and $\csc\theta$ is positive, what are the values of the other five trigonometric functions? (Draw a reference triangle)

2.3 Exact Values for Special Angles and Real Numbers

1. Sketch a reference triangle and find the reference angle for $\theta = -\pi/4$.

2. Find the exact value of $\cos -3\pi/2$.

3. Find the exact value of $\cos 7\pi/4$.

4. Find the least positive θ such that $\cos\theta = \frac{1}{\sqrt{2}}$, in both degrees and radians.

5. Find the exact value of all the angles between 0 and 2π radians for which $\cos\theta = -\sqrt{3}2$.

3 GRAPHING TRIGONOMETRIC FUNCTIONS

3.1 Basic Graphs

1. What are the x intercepts for the graph of $\sin x$ over the interval $-2\pi \leq x \leq 2\pi$?

2. Make a sketch of $y = \cos x$ for $-2\pi \leq x \leq 2\pi$ without using a calculator. Label each point where the graph crosses the x-axis in terms of π.

3. (a) Graph $y = A\cos x$ with viewing window $(-2\pi \leq x \leq 2\pi, -3 \leq y \leq 3)$ for $A = -3, 1$, and 2, all in the same viewing window.
   (b) Do the x-intercepts change? If so, where?
   (c) How far does each graph deviate from the x-axis? Experiment with other values of A.
   (d) Describe how the graph of $y = \cos x$ is changed by changing the value of A in $y = A\cos x$.

4. (a) Graph $y = \sin Bx$ with viewing window $(-\pi \leq x \leq \pi, -2 \leq y \leq 2)$ for $B = 1, 2$, and 3, all in the same viewing window.
   (b) How many periods of each graph appear in this viewing window. Experiment with other positive values of B.
   (c) How far does each graph deviate from the x-axis? Experiment with other values of A.

(d) Based on your experiments, how many periods of the graph $y = \sin nx$ for n a positive integer, would appear in this viewing window?

5. (a) Graph $y = \cos(x + C)$ with viewing window $(-2\pi \leq x \leq 2\pi, -2 \leq y \leq 2)$ for $C = -\pi/2, 0$, and $\pi/2$, all in the same viewing window. Experiment with other values of C.

   (b) Describe how the graph of $y = \cos x$ is changed by changing the value of C in $y = \cos(x + C)$.

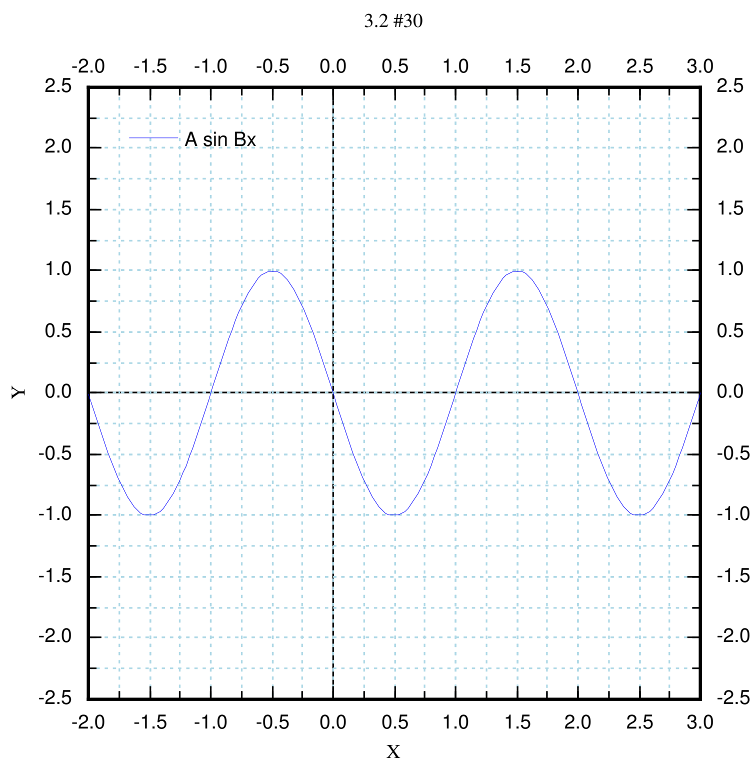
3.2 Graphing $y = k + A \sin Bx$ and $y = k + A \cos Bx$.

1. State the amplitude and period for the equation $y = -3 \cos x$, and graph it over the interval $0 \leq x \leq 4\pi$.

2. State the amplitude and period for the equation $y = 3 \cos 2x$, and graph it over the interval $-\pi \leq x \leq \pi$.

3. y is the displacement of an oscillating object from a central position at time t. Find an equation of the form $y = A \sin Bt$ or $y = A \cos Bt$ to satisfy the conditions that displacement from the t axis is $0yd$ when $t = 0$ the amplitude is $5ft$, and the period is $2sec$.

4. Find an equation of the form $y = A \sin Bx$ that produces this graph.



5. The equation $y = -4\cos 8t$, where t is time in seconds, represents the motion of a weight hanging on a spring after it has been pulled 4 cm below its equilibrium point and released (see the figure). What are the amplitude, period, and frequency of the function? [Air resistance and friction (damping forces) are neglected.] Graph the function for $0 \le t \le 3\pi/4$.

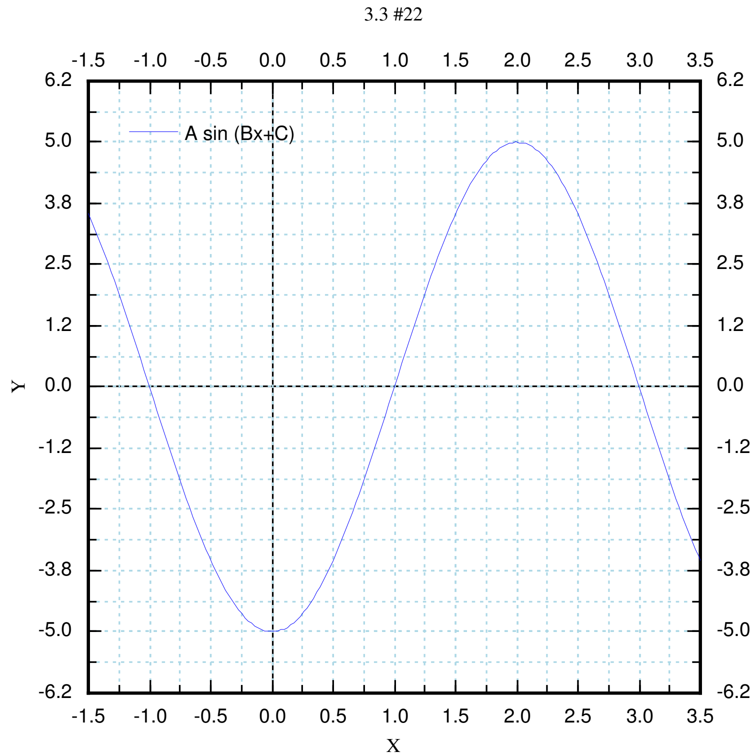
3.3 Graphing $y = k + A\sin(Bx + C)$ and $y = k + A\cos(Bx + C)$.

1. Indicate the phase shift for $y = \cos(x - \pi/2)$ and graph it over $\pi/2 \le x \le 5\pi/2$.

2. State the amplitude, period, and phase shift for $y = -3\sin(4x - \pi)$ and graph it over $-\pi \le x \le \pi$.

3. Graph $y = 4 - 3\sin(4x - \pi)$ over $-\pi \le x \le \pi$.

4. If the graph is a graph of an equation of the form $y = A\sin(Bx+C)$ with $-2 < C/B < 0$, find the equation.



5. State the amplitude, period, and phase shift for $y = -4\cos(4x + \pi/2)$ and graph it over $-\pi/2 \leq x \leq \pi$.

4 IDENTITIES

4.1 Fundamental Identities and Their Use

1. Use the fundamental identities to find the exact values of the remaining trigonometric functions of x, given that $\cos x = \frac{\sqrt{7}}{4}$ and $\cot x = \frac{-\sqrt{7}}{3}$.

2. Use the fundamental identities to find the exact values of the remaining trigonometric functions of x, given that $\cot x = -3/2$ and $\csc x > 0$.

3. Using fundamental identities, write $\sin y - \frac{\tan(-y)}{\sec y}$ in terms of sines and cosines and then simplify.

4. Using fundamental identities, write $\csc\theta\sec\theta - \csc\theta\cos\theta$ in terms of sines and cosines and then simplify.

4.2 Verifying Trigonometric Identities

1. Verify $\frac{\cos\alpha-\sin\alpha}{\sin\alpha\cos\alpha} = \csc\alpha - \sec\alpha$.

2. Verify $\frac{\cos^2\beta}{1+\sin\beta} = 1 + \sin\beta$.

3. Use a graphing calculator to see whether $\frac{\tan x}{\sin x+2\tan x} = \frac{1}{\cos x-2}$ appears to be an identity. If it is, verify it. If it is not, find a value of x for which both sides are defined but not equal.

4. Verify $\frac{\tan\beta}{\sec\beta-1} = \frac{\sec\beta+1}{\tan\beta}$.

4.3 Sum, Difference, and Cofunction Identities

1. Verify $\tan(x+\pi) = \tan x$ using a sum identity.

2. Find $\tan(x-y)$ if $\sin x = 1/5,\ \cos y = 2/5, \tan x < 0$, and $\tan y > 0$.

3. Verify $\frac{\cot\alpha+\cot\beta}{\cot\alpha-\cot\beta} = \frac{\sin(\beta+\alpha)}{\sin(\beta-\alpha)}$.

4. How would you show that $\sec(x+y) = \sec x + \sec y$ is not an identity?

4.4 Double-Angle and Half-Angle Identities

1. Verify $(\sin x + \cos x)^2 = 1 + \sin 2x$.

2. Verify $\cos(2t) = \frac{1-\tan^2 t}{1+\tan^2 t}$.

3. If $\cos(2x) = -\frac{28}{53}$ and $\pi/4 < x < \pi/2$, find $\sin x, \cos x$, and $\tan x$. Check your answers with a calculator.