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RESEARCH INTERESTS

Real and complex analysis, geometric function theory, conformal dynamics, probability theory, numerical analysis, analysis on fractals, quasiconformal geometry, computational geometry.

Some of my more particular interests have included: potential theory, fractal properties of harmonic measure, geometric properties of Brownian motion and other random processes, algebras generated by harmonic and holomorphic functions, geometry of hyperbolic manifolds and their covering groups, numerical computation of conformal mappings, multipole methods, optimal meshing algorithms and dimension distorting properties of quasiconformal maps. I program in C, Mathematica and Matlab as part of my investigations in these areas.

PROFESSIONAL EXPERIENCE

Sept. 1997 to present: Professor at SUNY, Stony Brook.
Sept. 1992 to Aug. 1997: Assoc. professor at SUNY, Stony Brook.
Sept. 1991 to Aug. 1992: Asst. professor at SUNY, Stony Brook.
Sept. 1988 to Aug. 1991: Hedrick Asst. professor at UCLA.
Sept. 1987 to Aug. 1988: NSF postdoc at MSRI, Berkeley.

EDUCATION

University of Chicago, Mathematics, Ph. D., 1987, Advisor Peter W. Jones.
Visiting graduate student and programmer, Dept. of Mathematics, Yale University, 1985-1987.
University of Chicago, Mathematics, Master of Science, 1984.
Cambridge University, Certificate of Advanced Study (Part III of Math. Tripos), 1983.
Michigan State University, Mathematics, Bachelor of Science, 1982.

PH.D. STUDENTS

Zsuzsanna Gonye, Ph. D. 2001, Geodesics in hyperbolic manifolds

Karyn Lundberg, Ph. D. 2005, Boundary behavior of conformal mappings

Hrant Hakobyan, Ph.D. 2007, Hausdorff dimension and quasisymmetric mappings

AWARDS

1992 Alfred P. Sloan Research Fellow

1987 NSF Postdoctoral Fellowship

1983-1986 McCormick Fellowship, NSF Fellowship, U. of Chicago

1982-1983 Churchill Fellowship, Cambridge England.

PUBLICATIONS

[1] C. J. Bishop. A counterexample in conformal welding concerning Hausdorff dimension. *Michigan Math. J.*, 35(1):151–159, 1988.

I construct an example of a closed Jordan curve with the property that harmonic measure (i.e., first hitting distribution of Brownian motion) for the two sides are comparable for every subset, but the curve has Hausdorff dimension > 1. This showed that even strong conditions on harmonic measure could not be used to characterize rectifiable curves.

[2] C. J. Bishop. An element of the disk-algebra that is stationary on a set of positive length. *Algebra i Analiz*, 1(3):83–88, 1989.

This answered a question of Havin and Makarov, by constructing an example of a non-constant analytic function on the unit disk whose derivative extends to be zero on a positive length subset of the unit circle.

[3] C. J. Bishop. Constructing continuous functions holomorphic off a curve. *J. Funct. Anal.*, 82(1):113–137, 1989.

This gives an explicit construction of certain space filling curves that arise naturally in function theory (they were previously known to exist by a result of Browder and Wermer using an indirect Hahn-Banach argument, but no one had "seen" one before).

[4] C. J. Bishop. Approximating continuous functions by holomorphic and harmonic functions. *Trans. Amer. Math. Soc.*, 311(2):781–811, 1989.

Various results are proven about algebras generated by harmonic functions on plane domains which generalize well known results on the unit disk. For example, the closed algebra generated by the bounded holomorphic functions and the complex conjugate of a single non-constant holomorphic function contains every uniformly continuous function on the domain.

[5] C. J. Bishop, L. Carleson, J. B. Garnett, and P. W. Jones. Harmonic measures supported on curves. *Pacific J. Math.*, 138(2):233–236, 1989.

We geometrically characterize when the harmonic measure from two opposite sides of a closed curve are either mutually singular or mutually continuous. For example, if we start

Brownian motions on opposite sides of fractal curve such as the von Koch snowflake, there are two disjoint sets which absorb the paths, i.e., the two hitting measures are mutually singular. I am fond of this paper because it gives me a co-authorship with Lennart Carleson.

[6] C. J. Bishop. Bounded functions in the little Bloch space. *Pacific J. Math.*, 142(2):209–225, 1990.

Sarason gave a non-constructive proof that there exist Blaschke products in the little Bloch space (a class of holomorphic functions on the disk whose derivatives grow slowly near the boundary) but no explicit example was known. This paper gives such an example and characterizes all such examples in terms of their zero sets. This result has been extended to various spaces by others.

[7] C. J. Bishop. Conformal welding of rectifiable curves. *Math. Scand.*, 67(1):61–72, 1990.

This answers a question of Walter Hayman. It constructs an example of a rectifiable curve that has a certain pathological property with respect to harmonic measure.

[8] C. J. Bishop and P. W. Jones. Harmonic measure and arclength. *Ann. of Math. (2)*, 132(3):511–547, 1990.

This paper proves a conjecture of Øksendal that has various consequence in geometric function theory. We show that a set of zero length which lies on a rectifiable curve has zero harmonic measure in any simply connected domain. It is one of my (and Peter's) most important and most quoted papers.

[9] C. J. Bishop and T. Steger. Three rigidity criteria for PSL$(2, \mathbf{R})$. *Bull. Amer. Math. Soc. (N.S.)*, 24(1):117–123, 1991.

A summary of my Acta. Math. paper with Steger. See [13] below.

[10] C. J. Bishop. A characterization of Poissonian domains. *Ark. Mat.*, 29(1):1–24, 1991.

A domain is called Poissonian if every bounded harmonic function on the domain is the Poission extension of a bounded function on the boundary. This paper provides a characterization of such domains in terms of the geometry of the boundary. It also proves some bounds on the complexity of the Martin boundary of any Euclidean domain. Conjectures made in this paper led to work of Tsirelson on the impossibility of stochastic processes with certain symmetries.

[11] C. J. Bishop. Brownian motion in Denjoy domains. *Ann. Probab.*, 20(2):631–651, 1992.

This considers a Cauchy process on the real line (which can be obtained from a Brownian motion in the plane restricted to the times when it is on the line). We show that for sets E of positive length, such a process has a well defined "direction of approach" to its first hitting of E. This answers a question of Chris Burdzy.

[12] C. J. Bishop. Some questions concerning harmonic measure. In *Partial differential equations with minimal smoothness and applications (Chicago, IL, 1990)*, volume 42 of *IMA Vol. Math. Appl.*, pages 89–97. Springer, New York, 1992.

A collection of open problems.

[13] C. J. Bishop and T. Steger. Representation-theoretic rigidity in PSL$(2, \mathbf{R})$. *Acta Math.*, 170(1):121–149, 1993.

Mostow rigidity implies that any two lattice inclusions of an abstract group into a connected simple Lie group are equivalent, if that group is not $PSL(2, R)$ (a better known consequence is that the geometry of a 3-dimensional hyperbolic manifold is determined by the fundamental group). This famous result fails for $PSL(2, R)$ because a surface can carry many non-equivalent Riemann surface structures. However, we show that a version of Mostow's theorem is still true: two representations of $PSL(2, R)$ restricted to two lattices are equivalent iff they are the same representation and the two lattices are conjugate.

[14] C. J. Bishop. An indestructible Blaschke product in the little Bloch space. *Publ. Mat.*, 37(1):95–109, 1993.

Another explicit construction of a holomorphic function with odd properties.

[15] C. J. Bishop. How geodesics approach the boundary in a simply connected domain. *J. Anal. Math.*, 64:291–325, 1994.

This answers a question of Chris Burdzy about the geometric properties of a hyperbolic geodesics in a plane domain. In particular, if a geodesics passes within a small Euclidean distance of a boundary point, does it have to hit the boundary near this point? If not, how far away can the hitting point be on average? The most amusing aspect of this paper is that the answer involves the convergence or divergence of an integral of the form $\int_0^1 f(t)^{2/9} dt/t$.

[16] C. J. Bishop and P. W. Jones. Harmonic measure, L^2 estimates and the Schwarzian derivative. *J. Anal. Math.*, 62:77–113, 1994.

This simplifies and extends results from [8] and is part of a broader program to use "L^2 techniques" on sets rather than functions. Peter and I develop an analog of Littlewood-Paley theory based on the Schwarzian derivative (in place of the usual derivative) and use it to prove a number of conjectures from function theory, including a geometric characterization of "BMO domains" and an a.e. characterization of tangent points of a curve. Surprisingly, this paper is now cited in various papers dealing with large data sets because of a lemma that characterizes when a set in the plane can be approximated by a curve of finite length. This result was later generalized by others to estimate how well a set of points in a high dimensional space can be approximated by a lower dimensional (but not necessarily linear) submanifold.

[17] C. J. Bishop. Some homeomorphisms of the sphere conformal off a curve. *Ann. Acad. Sci. Fenn. Ser. A I Math.*, 19(2):323–338, 1994.

This gives some examples of some unusual homeomorphism of the 2-sphere to itself.

[18] C. J. Bishop. A counterexample concerning smooth approximation. *Proc. Amer. Math. Soc.*, 124(10):3131–3134, 1996.

This answers a question of Stegenga and Smith about approximations in Sobolev spaces.

[19] C. J. Bishop. A distance formula for algebras on the disk. *Pacific J. Math.*, 174(1):1–27, 1996.

This gives a characterization of certain algebras of functions on the unit disk which are generated by harmonic and holomorphic functions.

[20] C. J. Bishop. Minkowski dimension and the Poincaré exponent. *Michigan Math. J.*, 43(2):231–246, 1996.

This gives a method of estimating the Hausdorff dimension of fractals known as Kleinian limit sets. The techniques here were adapted by others to some Julia sets.

[21] C. J. Bishop. On a theorem of Beardon and Maskit. *Ann. Acad. Sci. Fenn. Math.*, 21(2):383–388, 1996.

This gives a characterization of geometrically finite Kleinian group. It is a cleaner version of an earlier result by Beardon and Maskit.

[22] C. J. Bishop. Some characterizations of $C(\mathcal{M})$. *Proc. Amer. Math. Soc.*, 124(9):2695–2701, 1996.

This gives a simple characterization of which continuous functions on the unit disk are in the algebra generated by all bounded harmonic functions. This happens iff the level sets of the function can be approximated by curves on which arc length is a Carleson measure.

[23] C. J. Bishop. Geometric exponents and Kleinian groups. *Invent. Math.*, 127(1):33–50, 1997.

This proves an old conjecture of Maskit about Kleinian groups.

[24] C. J. Bishop and Y. Peres. Packing dimension and Cartesian products. *Trans. Amer. Math. Soc.*, 348(11):4433–4445, 1996.

This gives optimal estimates for the dimension of the product of two sets. Previous results gave bounds which might or might not be sharp for a particular set, but this gives the sharp result for any set.

[25] C. J. Bishop and Peter W. Jones. Hausdorff dimension and Kleinian groups. *Acta Math.*, 179(1):1–39, 1997.

This proves three main results: the critical exponent of a Kleinian group equals the Hausdorff dimension of its radial limit set, the limit set of a finitely generated Kleinian group has Hausdorff dimension two iff the group is geometrically infinite, and Hausdorff dimension of the limit set is upper semi-continuous. Each of these was a well known conjecture in the field. This is the most cited paper for both Peter Jones and myself. Our results have been extended to other settings such as symmetric spaces, variable curvature, Gromov hyperbolic groups and discrete quasiconformal groups.

[26] C. J. Bishop and P. W. Jones. The law of the iterated logarithm for Kleinian groups. In *Lipa's legacy (New York, 1995)*, volume 211 of *Contemp. Math.*, pages 17–50. Amer. Math. Soc., Providence, RI, 1997.

We prove a conjecture of Dennis Sullivan regarding the exact Hausdorff dimension of certain Kleinian group limit sets by exploiting the connections between harmonic functions and martingales. We also answer a question of Curt McMullen regarding absolute continuity of conjugations between certain types of groups.

[27] C. J. Bishop and P. W. Jones. Wiggly sets and limit sets. *Ark. Mat.*, 35(2):201–224, 1997.

We show that if a connected set deviates from a line segment at all points and all scales, then it must have dimension strictly larger than 1. The proof is a rather intricate argument involving the so called "traveling salesman theorem" of Peter Jones, but so far no one has found an easier proof of this easy looking result.

[28] C. J. Bishop, P. W. Jones, Robin Pemantle, and Yuval Peres. The dimension of the Brownian frontier is greater than 1. *J. Funct. Anal.*, 143(2):309–336, 1997.

We prove that the frontier of a Brownian motion (i.e., the boundary of its unbounded complementary component) has Hausdorff dimension strictly bigger than one, providing the first evidence for a conjecture of Mandelbrot that the dimension equals 4/3. The full conjecture was later proven using Stochastic Lowner Evolutions (SLE) as developed by Lawler, Schramm and Wendelin (resulting in Fields medal for the latter). One fun aspect of our proof is a nested decomposition of the plane into fractal tiles, instead of the usual square grid (we needed to avoid straight lines in certain boundary estimates).

[29] C. J. Bishop. Quasiconformal mappings which increase dimension. *Ann. Acad. Sci. Fenn. Math.*, 24(2):397–407, 1999.

This answers a question of Heinonen by showing that the dimension of any set $E \subset R^n$ with $0 < \dim(E) < n$ can be increased by taking a quasiconformal image. This is interesting because the dimension cannot always be lowered (understanding why is currently a big part of analysis on metric spaces).

[30] C. J. Bishop. A quasisymmetric surface with no rectifiable curves. *Proc. Amer. Math. Soc.*, 127(7):2035–2040, 1999.

Builds a pathological surface in 3-space that is nice in certain respects, but which contains no connected set of finite length. This answered a question of Rohde. This result was extended by David and Toro.

[31] C. J. Bishop, A. Böttcher, Yu. I. Karlovich, and I. Spitkovsky. Local spectra and index of singular integral operators with piecewise continuous coefficients on composed curves. *Math. Nachr.*, 206:5–83, 1999.

This discusses the properties of a certain class of operators. My contribution was limited to the proof of certain estimates regarding harmonic measure.

[32] C. J. Bishop and J. T. Tyson. Conformal dimension of the antenna set. *Proc. Amer. Math. Soc.*, 129(12):3631–3636 (electronic), 2001.

We compute the minimal possible dimension over all possible quasiconformal images of a certain fractal. In trying to understand how the dimension of a set can be lowered by QC mappings, there are very few non-trivial cases where we actually know what the minimum is. This provides one such example.

[33] C. J. Bishop and J. T. Tyson. Locally minimal sets for conformal dimension. *Ann. Acad. Sci. Fenn. Math.*, 26(2):361–373, 2001.

This gives more examples relevant to dimension lowering.

[34] C. J. Bishop. Bi-Lipschitz homogeneous curves in $\mathbb{R}^2$ are quasicircles. *Trans. Amer. Math. Soc.*, 353(7):2655–2663 (electronic), 2001.

This proves that if a Jordan curve is biLipschitz homogeneous then it is "nice" (i.e., satisfies Ahlfors' 3-point condition). Homogeneous means that any point in the set can be mapped to any other point by a map which is biLipschitz on the set. This had been an open problem in the field.

[35] C. J. Bishop. Divergence groups have the Bowen property. *Ann. of Math. (2)*, 154(1):205–217, 2001.

This completes a series of papers by Rufus Bowen, Dennis Sullivan, Kari Astala and Michel Zinsmeister by showing that a Fuchsian group is divergence type iff it has Bowen's property (this says that any deformation of the group has a limit set which is either a circle or has Hausdorff dimension > 1).

[36] C. J. Bishop. BiLipschitz approximations of quasiconformal maps. *Ann. Acad. Sci. Fenn. Math.*, 27(1):97–108, 2002.

This is a technical result that shows quasiconformal maps in two dimensions can be approximated by bi-Lipschitz maps in a certain precise sense.

[37] C. J. Bishop. Quasiconformal mappings of Y-pieces. *Rev. Mat. Iberoamericana*, 18(3):627–652, 2002.

A technical result that discusses the optimal way to deform a Riemann surface. It is used in the following paper.

[38] C. J. Bishop. Non-rectifiable limit sets of dimension one. *Rev. Mat. Iberoamericana*, 18(3):653–684, 2002.

Answers a question of Astala and Zinsmeister by constructing certain deformations of Fuchsian groups whose limit sets are non-rectifiable curves of dimension 1.

[39] C. J. Bishop and P. W. Jones. Compact deformations of Fuchsian groups. *J. Anal. Math.*, 87:5–36, 2002. Dedicated to the memory of Thomas H. Wolff.

We show that a deformation of a Fuchsian group which only changes the conformal structure on a compact set gives a limit set where the escaping limit set had dimension 1.

[40] C. J. Bishop. Quasiconformal Lipschitz maps, Sullivan's convex hull theorem and Brennan's conjecture. *Ark. Mat.*, 40(1):1–26, 2002.

This gives a new proof of a result of Dennis Sullivan about convex sets in hyperbolic space and gives applications to conformal maps in the plane. One such is the fact that any simply connected plane domain can be mapped to a disk by a Lipschitz homeomorphism.

[41] C. J. Bishop, V. Ya. Gutlyanskiĭ, O. Martio, and M. Vuorinen. On conformal dilatation in space. *Int. J. Math. Math. Sci.*, (22):1397–1420, 2003.

This shows that if the dilatation of a quasiconformal map in space satisfies certain integrability conditions then the map is pointwise differentiable.

[42] C. J. Bishop. Big deformations near infinity. *Illinois J. Math.*, 47(4):977–996, 2003.

This proves the existence of certain deformations of a Fuchsian group. These are used in [48] to study the analytic dependence of a limit set as a function of a deformation parameter.

[43] C. J. Bishop. δ-stable Fuchsian groups. *Ann. Acad. Sci. Fenn. Math.*, 28(1):153–167, 2003.

This introduces the idea of a δ-stable group and gives examples of such things.

[44] C. J. Bishop. An explicit constant for Sullivan's convex hull theorem. In *In the tradition of Ahlfors and Bers, III*, volume 355 of *Contemp. Math.*, pages 41–69. Amer. Math. Soc., Providence, RI, 2004.

This gives the best currently known constant ($K = 7.82$) in a theorem of Sullivan. The size of this constant has implications for planar conformal mappings.

[45] C. J. Bishop. The linear escape limit set. *Proc. Amer. Math. Soc.*, 132(5):1385–1388 (electronic), 2004.

Proves some results concerning the Hausdorff dimension of special limit sets.

[46] C. J. Bishop. Orthogonal functions in H^∞. *Pacific J. Math.*, 220(1):1–31, 2005.

This disproves a well known conjecture of Walter Rudin: if f is holomorphic on the unit disk and the sequence of powers $f, f^2, f^3, \ldots$ are orthogonal, must f be an inner function (i.e., $|f| = 1$ a.e. on the circle)? An independent solution was obtained by Carl Sundberg.

[47] C. J. Bishop. Boundary interpolation sets for conformal maps. *Bull. London Math. Soc.*, 38(4):607–616, 2006.

This characterizes interpolating sets for conformal maps of the disk, i.e., sets E on the circle so that given any homeomorphism we can find a conformal map on the disk whose restriction to E agrees with the given map.

[48] C. J. Bishop. A criterion for the failure of Ruelle's property. *Ergodic Theory Dynam. Systems*, 26(6):1733–1748, 2006.

We give examples to show that even for divergence type groups, the dimension of the limit set need not be an analytic function of the deformation.

[49] C.J. Bishop. Harmonic measure. *Book review in Bull. Amer. Math. Soc.* 44(2):267-276, 2007.

This is an expository survey of recent results in geometric function theory.

[50] C.J. Bishop. An A_1 weight not comparable to any quasiconformal Jacobian. *In the tradition of Ahlfors-Bers, IV*, volume 432 of *Contemp. Math.*, pages 7–18. Amer. Math. Soc., Providence, RI. 2007

Gives the example described by the title. This solved a problem of Stephen Semmes. Characterizing the Jacobians of quasiconformal maps has been one of the driving problems in the field, and this example shows that no sufficient condition only in terms of the distribution

function is possible. I also construct a surface in Euclidean 3-space which is quasisymmetrically equivalent to the plane, but which is not bi-Lipschitz equivalent to the plane. No previous example was even embeddable in a Banach space.

[51] C.J. Bishop and H. Hakobyan. A central set of dimension 2. 2007. to appear in Proc. Amer. Math. Soc.

The medial axis of a domain is the set of centers of disks in the domain which hit the boundary in two or more points. The central set is the set of centers of maximal disks in the domain. They are distinct sets in general, but are sometimes identified by mistake in the literature. Here we emphasize the difference by constructing a Lipschitz domain where the medial axis has dimension 1 and the central set has dimension 2.

[52] C.J. Bishop. Conformal welding and Koebe's theorem. *Ann. of Math.* 166(2): 613–656, 2007.

A conformal welding is a homeomorphism of the unit circle to itself of the form $h = g^{-1} \circ f$ where f and g are conformal maps of the two sides of the circle to two sides of a closed Jordan curve. It is known that not every homeomorphism has this form, but we prove that every homeomorphism is "almost" a conformal welding in a precise sense. This paper also contains a new and simple proof of the famous result that every quasisymmetric circle mapping is a conformal welding.

[53] C.J. Bishop. Decreasing dilatations can increase dimension. 2006. to appear in the Illinois J. of Math.

Answers a question of Zinsmeister about quasiconformal maps and dimension.

PREPRINTS

[54] C.J. Bishop. Non-removable sets for quasiconformal and quasi-isometric mappings in $\mathbb{R}^3$.

We construct a Cantor set in 3-space which is not removable for QC mappings. This is the only known non-trivial example of a non-removable set in dimension higher than 2.

[55] C.J. Bishop. A set containing rectifiable arcs locally but not globally.

This gives examples related to characterizing the Jacobians of quasiconformal maps. The paper [50] shows that geometry of the set where a Jacobian blows up is crucially important and this paper gives examples that must be avoided.

[56] C.J. Bishop. Conformal mapping in linear time.

Gives an algorithm for computing conformal maps. The motivation for this method comes from hyperbolic manifolds and computational geometry. Moreover, the method is near optimal. It will find an ϵ approximation to the conformal map onto an n-gon in time $O(n|\log \epsilon \log\log \epsilon|)$. The techniques applied come from hyperbolic 3-manifolds, computational geometry, numerical analysis and geometric function theory. I would like to think this is one of my best results.

[57] C.J. Bishop. Distortion of disks by conformal maps.

Answers a question of Astala about how a collection of disks can be distorted by a QC map which is conformal outside the disks.

[58] C.J. Bishop. Bounds for the CRDT algorithm.

In 1998 Driscoll and Vavasis introduced the CRDT algorithm for computing conformal maps. Although it works well in practice, there is no proof that it converges to the correct answer. We partially address this by showing the CRDT algorithm will always give an answer which is close to the correct answer in a certain precise sense (with bounds independent of the domain)

[59] C.J. Bishop. A fast quasiconformal mapping theorem for polygons.

Gives a linear time approximation to the Riemann map. This is an introduction to the longer paper "Conformal mapping in linear time".

[60] C.J. Bishop. Optimal angle bounds for quadrilateral meshes.

This shows that any polygon has a linear size quadrilateral mesh so that every new angle used is between 60 and 120 degrees. The angle bounds are optimal. This answers a question of Bern and Eppstein. Previously, it was not known if there was a linear size mesh with new angles bounded from below.

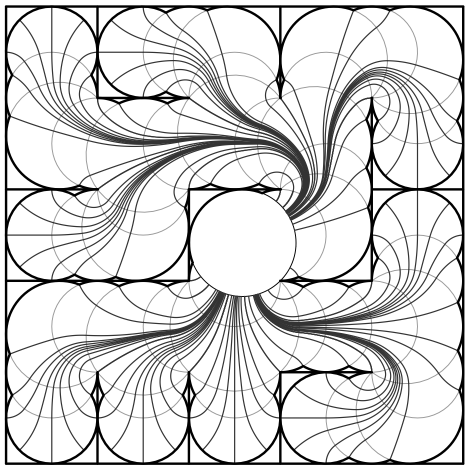
[61] C.J. Bishop. Tree-like decompositions and conformal maps.

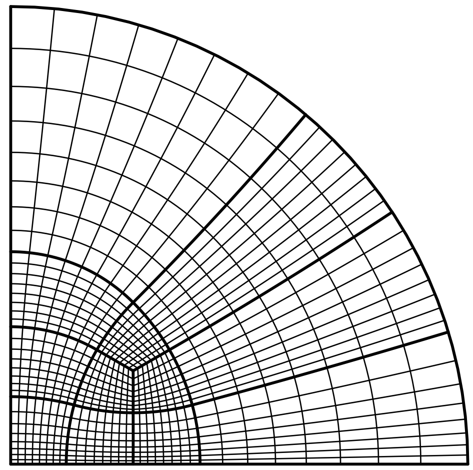
Shows that any simply connected planar domain can be decomposed into "roundish" pieces efficiently. An application to approximating conformal maps is discussed. This answers a question of Steve Vavasis.

[62] C.J. Bishop. Estimates for harmonic conjugation.

Gives a geometric characterization of the planar domains for which harmonic conjugation defines a L^2 bounded operator on the boundary. The answer is in terms on a decomposition of the domain into pieces and roughly says that conjugation is bounded iff the Poincaré inequality holds on a weighted tree related to the decomposition.

POSTSCRIPT





Some illustrations from my recent work on numerical analysis and computational geometry. On the left the curved lines illustrate a flow from the boundary of a domain to an interior circle. This flow has a simple geometric definition (inspired by a result of Dennis Sullivan on hyperbolic 3-manifolds), but gives a uniform approximation to the Riemann mapping, with estimates independent of the domain and forms the first step of an algorithm that converges quadratically to the conformal map. On the right is a quadrilateral meshing of a circular triangle which forms a "worst case" in my algorithm which finds a quadrilateral mesh of any polygon with all new angles between $60°$ and $120°$. These angle bounds are best possible and so is the required time, $O(n)$ for an n-gon.

Copies of my papers and lectures are available at `www.math.sunysb.edu/~bishop`