

SAMPLE FINAL MAT 142
FINAL is Thursday, May 11, 2000,
12-3pm
Psych A-137

1. Place the letter corresponding to the correct answer in the box next to each question.

- (i) The sequence $\{a_n\} = \{1 + (-1)^n \frac{1}{n}\}$ converges to
(a) 0 (b) -1 (c) 1 (d) $\frac{1}{2}$ (e) $-\frac{1}{2}$ (f) it diverges
- (ii) The sequence $\{a_n\} = \{(-1)^n(1 - \frac{1}{n})\}$ has least upper bound equal to
(a) -1 (b) 0 (c) 1 (d) 2 (e) $\frac{1}{2}$ (f) it has no upper bound
- (iii) Define a sequence by $a_0 = 1$, $a_n = \frac{3}{2}a_{n-1}$. Then the sequence converges to
(a) 0 (b) 1 (c) 2 (d) 4 (e) $\frac{3}{2}$ (f) the sequence diverges
- (iv) The infinite series $\sum_{n=0}^{\infty} 3^{-n}$ converges to
(a) 0 (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$ (e) 1 (f) none of these
- (v) The infinite series $\sum_{n=0}^{\infty} \frac{2^n}{n!}$ converges to
(a) 0 (b) 1 (c) e (d) e^2 (e) 2 (f) none of these
- (vi) What is the inverse function of $y = \sqrt{1 - x^2}$ on $(0, 1)$?
(a) $x^2 - 1$ (b) $\sqrt{1 - x^2}$ (c) $x^2 + 1$ (d) $\sqrt{1 + x^2}$ (e) $\sqrt{1 - x}$ (f) none of these
- (vii) $\int_0^2 \frac{2x}{x^2-5} dx =$
(a) $\ln 2$ (b) $\ln 5$ (c) $-\ln 5$ (d) $-\ln 2$ (e) 0 (f) none of these
- (viii) $\frac{d}{dx} 2^{x^2} =$
(a) 2^{x^2} (b) $2^{x^2} 2x$ (c) $2^{x^2} \ln 2$ (d) $2^{x^2} 2x \ln 2$ (e) $2^{x^2} x^2 \ln 2$ (f) none of these
- (ix) Find the limit $\lim_{x \rightarrow 0^+} (1 + x)^{1/x}$.
(a) 0 (b) $1/e$ (c) 1 (d) e (e) ∞ (f) none of these
- (x) What is $\frac{d}{dx} \sin^{-1}(x), |x| < 1$?
(a) $x/\sqrt{1+x^2}$ (b) $1/\sqrt{1+x^2}$ (c) $1/\sqrt{1-x^2}$ (d) $-1/\sqrt{1-x^2}$ (e) $1/(|x|\sqrt{x^2-1})$ (f) none of these

2. Evaluate each of the following integrals. You may use the table of integrals at the end of the book.

(i) $\int \sin^3(x) dx$

(ii) $\int \frac{dx}{1+\sin 3x}$

(iii) $\int \sqrt{x^2 - 1} dx$

(iv) $\int \frac{dx}{\sqrt{4+x^2}}$

(v) $\int \frac{\sqrt{x+2}}{x} dx$

3. Put a “C” in the box if the series converges and a “D” if it diverges.

(i) $\sum_{n=1}^{\infty} \frac{\sin n}{n^3}$

(ii) $\sum_{n=1}^{\infty} \frac{1}{1+n^2}$

(iii) $\sum_{n=1}^{\infty} (-1)^n n^{1/2}$

(iv) $\sum_{n=1}^{\infty} \frac{n^4}{2^n}$

(v) $\sum_{n=1}^{\infty} (-1)^n n^2 1 + n^3$

4. Write out the Maclaurin series for $\sin x$ and $\tan x$ up to the third power. Use these series to evaluate

$$\lim_{x \rightarrow 0} \frac{\sin x - \tan x}{x^3}.$$

5. Prove that

$$1 + \frac{1}{2} \cdot 2 + \frac{1}{4} \cdot 3 + \dots + \frac{n}{2^{n-1}} + \dots = 4.$$

(Hint: differentiate the equation $\frac{1}{1-x} = 1 + \sum_{n=1}^{\infty} x^n$ and plug in an appropriate value.)

6. Give the definition of the hyperbolic functions $\sinh x$ and $\cosh x$. Using these definitions, show $\sinh 2x = 2 \sinh x \cosh x$.