SAMPLE FINAL, MAT 131, Spring 2001

The final will be on Friday, May 11 from 11:00am pm to 1:30 pm. Room assignments are the same as for the first midterm.

Location	Sections	Proctors
Javits 109	1,2	Kumple, Unal*
Javits 110	3,4,5,6	Minski, Han*, Robles, Longoni
Javits 111	7,8	Sung*, Kim

The following gives a rough idea of the type of problems and topics covered on the midterm but it is not an exact match. The number and type of problems may be different on the actual exam.

1. Evaluate each of the following integrals.

   (a) $\int_0^1 x^{10} + x^{1/2} dx$

   (b) $\int_0^\pi \sin(x) dx$

   (c) $\int_0^2 e^x + \frac{1}{x} dx$

   (d) $\int \cos(x^2) x dx$

   (e) $\int \sqrt{3x+1} dx$

   (f) $\int_{-1}^1 \sin(x^3) dx$

2. Place the letter corresponding to the correct answer in the box next to each question. Each correct answer is worth 2 points.

   (i) ☐ The definition of $f'(x)$ is
   (a) $\lim_{h\to 0} \frac{f(x+h)-f(h)}{h}$ **(b)** $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ **(c)** $\lim_{x\to 0} \frac{f(x+h)-f(x)}{h}$ **(d)** $\lim_{h\to x} \frac{f(x+h)-f(x)}{h}$
   (e) $\lim_{h\to 0} \frac{f(x+h)-f(x)}{x}$ **(f)** none of these.

   (ii) ☐ Suppose $\int_0^{f(x)} t^2 dt = x^2$. Then $f(x) =$
   (a) x **(b)** $3^{1/3}x^{2/3}$ **(c)** x^2 **(d)** $2x^2$ **(e)** $x^{-2/3}$ **(f)** none of these.

   (iii) ☐ Find the equation of the curve in the xy-plane that passes through the point $(1,3)$ if its slope at x is always $3x^2+2$.
   (a) $y = x^3+2x$ **(b)** $y=5$ **(c)** $y = 3x^2-2$ **(d)** $y = x^3+2x-4$ **(e)** $x=-1$ **(f)** none of these.

   (iv) ☐ If $f(x) = x^3 - 3x$ then the iterative formula for Newton's method becomes $x_{n+1} =$
   (a) $x_n - x_n^3 - 3x_n$ **(b)** $x_n/(3x_n-3)$ **(c)** $x_n - (x_n^3-3x_n)/(3x_n^2-3)$
   (d) $x_n - (3x_n^2-3)/(x_n^3-3x_n)$ **(e)** $x_n - 3x_n^2+3$ **(f)** none of these.

   (v) ☐ Let $F(t) = \int_0^t (1+\sin^4(x))(1-x)dx$ Then F takes its maximum value at
   (a) 1 **(b)** 0 **(c)** $\pi/2$ **(d)** π **(e)** 2π **(f)** none of these.

(vi) ☐ What is the name of the following result: if f is continuous on an interval containing a and b and $f(a) < y < f(b)$ then there is a c with $a < c < b$ such that $f(c) = y$
(a) Rolle's theorem **(b)** mean value theorem **(c)** Riemann's theorem **(d)** min-max theorem **(e)** Simpson's rule **(f)** intermediate value theorem

(vii) ☐ What is $\lim_{x\to 2} \frac{x^2-4}{x-2}$? **(a)** 1 **(b)** 2 **(c)** 3 **(d)** 4 **(e)** ∞ **(f)** none of these.

(viii) ☐ A car drives 30 miles at 60 mph and then another 50 miles at 50 mph. What is the average speed for the entire trip?
(b) $52\frac{1}{2}$ mph **(c)** $53\frac{1}{3}$ mph **(d)** 55 mph **(e)** 57 mph **(f)** none of these.

(ix) ☐ On planet X a ball dropped from rest falls 36 meters in 2 seconds. The acceleration due to gravity is
(a) 36m/sec^2 **(b)** 18m/sec^2 **(c)** 6m/sec^2 **(d)** $\sqrt{18}$m/sec^2 **(e)** 3m/sec^2 **(f)** none of these.

(x) ☐ Find $\frac{dy}{dx}$ if $x^2y - y = 1$
(a) $\frac{dy}{dx} = \frac{-2xy}{x^2-1}$ **(b)** $\frac{dy}{dx} = \frac{2xy}{x^2-1}$ **(c)** $\frac{dy}{dx} = \frac{xy}{2y+x^2}$ **(d)** $\frac{dy}{dx} = \frac{-2xy}{y+x}$ **(e)** $\frac{dy}{dx} = (1-2xy)(x^2-1)$ **(f)** none of these.

(xi) ☐ Suppose $f'(x) = 1 - \sin^{10}(x)$. Then on the interval $[0, \frac{1}{2}\pi]$ the function f is
(a) increasing and concave down **(b)** increasing and concave up **(c)** decreasing and concave down **(d)** decreasing and concave up **(e)** constant **(f)** none of these.

(xii) ☐ Which of the following is an anti-derivative of x^3? **(a)** $3x^2$ **(b)** $3x^4$ **(c)** $\frac{1}{3}x^3$ **(d)** $\frac{1}{4}x^4$ **(e)** $\frac{1}{3}x^4$ **(f)** none of these.

(xiii) ☐ The quotient rule says that $(f/g)' =$ **(a)** f'/g' **(b)** $(f'g-fg')/g^2$ **(c)** $(f'g'-fg)/f^2$ **(d)** $fg - f'g'$ **(e)** $(fg' - f'g)/g^2$ **(f)** none of these.

3. A man starts walking north at 4 ft/s from a point P. Five minutes later a woman starts walking south at 5 ft/s from a point 500 ft east of P. At what rate are the people moving apart, 15 minuest after the woman starts walking?

4. Compute $\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} k$.

5. Find these limits by l'Hopitals' rule

   (i) $\lim_{x\to 1} \frac{x^2-1}{x-1}$

   (ii) $\lim_{x\to\infty} \frac{x^2}{e^x}$

   (iii) $\lim_{x\to 0} \frac{\ln x}{x}$

6. Find two positive numbers whose product is 100 and whose sum is a minimum.