

MAT 131, Calculus I Fall 2005
Sample Final Exam

(1) For each of the questions below, indicate if the statement is true (**T**) or false (**F**).

a	f
b	g
c	h
d	i
e	j

- (a) $f(x) = x \ln(x) - x$ is an antiderivative of $\ln x$ on $(0, \infty)$.
- (b) The horizontal axis is a horizontal asymptote for $f(x) = \frac{x^3}{3e^x}$.
- (c) $f(x) = (\sin x) \cdot e^x$ is concave upward on $(0, \pi/2)$.
- (d) If f, g are continuous on $(0, \infty)$ then $\int_1^2 f(t) \cdot g(t) dt = (\int_1^2 f(t) dt) \cdot (\int_1^2 g(t) dt)$.
- (e) $g(x) = \frac{3(x+1)(x-2)}{(x-7)(x-1)}$ is differentiable on $(-\infty, \frac{1}{2})$.
- (f) $\frac{d}{dx} \ln(1 + x^2) = \frac{2x}{(1+x^2)^2}$.
- (g) $f(x) = (x - 1)^5$ has one local minimum and one local maximum on $(-2, 2)$.
- (h) $\int_1^2 \frac{3\sqrt{x}+x^3}{x} = 6(\sqrt{2} - 1) + \frac{7}{3}$.
- (i) The tangent line to the graph of $f(x) = \sqrt{1 + x^3}$ at $(a = 2)$ has an equation given by $y = 2x - 1$.
- (j) $\int_{-2}^2 \frac{1}{t^4} dt = -\frac{1}{3}(\frac{1}{8} - \frac{1}{-8}) = -\frac{1}{12}$.

- (2) (a) Find $\lim_{x \rightarrow +\infty} x^2 \cdot \ln(1 + \frac{1}{x})$.
- (b) Find the derivative of $f(x) = (\cos 3x)^x + x \ln x - x$.
- (c) Compute $\lim_{x \rightarrow +\infty} (x + \frac{\cos x}{x^2}) - x \cdot (\sqrt{1 + \frac{1}{x}})$.
- (d) Find an antiderivative $F(x)$ of $f(x)$ on $(0, +\infty)$ when $f(x) = \sin(7x + 12) + \sqrt{2x}$.
- (e) Find $\frac{dy}{dx}$ when you know that $y^3 + 3xy + x^2 = 0$.
- (3) Let $f(x) = x \cdot (x - 2)^3$
- (a) Find the absolute maximum, the absolute minimum and the local ones on $[-3, 3]$.
- (b) Find the largest intervals included in $[-3, 3]$ where f is increasing, decreasing, and then concave upward.
- (4) Newton's method.
- (a) Write down the two first steps x_1, x_2 for Newton's method applied to the equation $f(x) = x^3 - 2$ with starting point $x_0 = 1$.
- (b) Redo the exercise from a previous homework 4 page 325.
- (5) (a) Compute $\int_1^5 \cos(3x + 12) + \frac{\sqrt{x+x^3}}{x^4} dx$.
- (b) Show that $\int_2^7 \sqrt{3 + x^2 + 7x^4} dx \geq 5\sqrt{3}$.
- (c) Find $\frac{d}{dx} \int_{-1}^{x^2} \sin(3 + t^2) dt$.
- (d) Compute $\int_0^1 \frac{7e^{3x}}{1+e^{3x}} dx$.
- (e) Compute $\int_0^{\pi/2} (\cos t \cdot \sin^3 t + 2 \sin t \cdot \cos t) dt$.
- (f) What is the following limit: $\lim_{n \rightarrow +\infty} \sum_{i=1}^n \frac{i^7}{n^8}$?
- (6) Among all the right angle triangles of fixed area (equal to $1/2$), find the one with the shortest hypotenuse.