

$$(15) f(x, y) = x^2 + xy + y^2 \quad ; \quad D = \{ (x, y) \mid x^2 + y^2 \leq 1 \}$$

Critical points: $\nabla f = \langle 2x + y, x + 2y \rangle = \langle 0, 0 \rangle$

$$\Leftrightarrow x = 0 \text{ and } y = 0$$

$\therefore (0, 0)$ is the only critical point.

$$f(0, 0) = 0$$

Boundary of D: Unit circle $\underbrace{x^2 + y^2 = 1}_{g(x, y)}$

- Lagrange Multipliers:

* Optimize $f(x, y) = x^2 + xy + y^2$ subject to the constraint $g(x, y) = x^2 + y^2 = 1$.

Solve the system of equations:
$$\begin{cases} \nabla f = \lambda \nabla g \\ g = 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} 2x + y = \lambda 2x \\ x + 2y = \lambda 2y \\ x^2 + y^2 = 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} y = x(2\lambda - 2) & (1) \\ x = y(2\lambda - 2) & (2) \\ x^2 + y^2 = 1 & (3) \end{cases}$$

$$(1) \ \& \ (2) \Rightarrow \frac{y}{x} = \frac{x}{y} \Rightarrow x^2 = y^2$$

$$(3) \Rightarrow \Downarrow x^2 = \frac{1}{2}, y^2 = \frac{1}{2}$$

$$\Rightarrow x, y = \pm \frac{1}{\sqrt{2}}$$

⑤