

$$(11) (e) f(x, y) = x \sin y$$

Critical points: $\nabla f = \langle \sin y, x \cos y \rangle = \langle 0, 0 \rangle$

$$\Leftrightarrow \sin y = 0 \quad \text{and} \quad x \cos y = 0.$$

↓

$$y = k\pi,$$

$$k = 0, \pm 1, \pm 2, \dots$$

(k any integer)

↙ ↘

$$x = 0 \text{ or}$$

$$\cos y = 0$$

this can't be the case since

$$y = k\pi \text{ and}$$

$$\cos(k\pi) = \begin{cases} 1 & \text{if } k \text{ even} \\ -1 & \text{if } k \text{ odd} \end{cases}$$

$$\therefore \{(0, k\pi) \mid k = 0, \pm 1, \pm 2, \dots\} = \text{Critical points of } f.$$

$$\begin{aligned} * \quad & \left. \begin{aligned} f_{xx} &= 0 \\ f_{yy} &= -x \sin y \\ f_{xy} &= \cos y \end{aligned} \right\} \Rightarrow D(x, y) = -\cos^2 y \\ & D(0, k\pi) = -\cos^2(k\pi) \\ & = -(\pm 1)^2 \\ & = -1 < 0 \end{aligned}$$

⇒ All critical points are saddles.