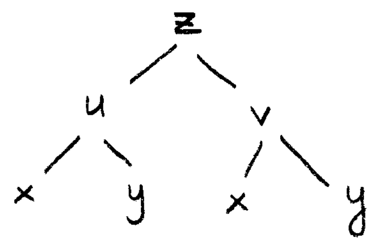
MAT 205 - Solutions to Selected Problems in Review Sheet II

(1) $z = f(u,v) = \dfrac{u^2+v^2}{u^2-v^2}$, $u(x,y) = e^{-x-y}$, $v(x,y) = e^{xy}$



$$\frac{\partial z}{\partial u} = \frac{\partial f}{\partial u} = \frac{2u(u^2-v^2) - 2u(u^2+v^2)}{(u^2-v^2)^2} = \frac{-4uv^2}{(u^2-v^2)^2}$$

$$\frac{\partial z}{\partial v} = \frac{\partial f}{\partial v} = \frac{2v(u^2-v^2) + 2v(u^2+v^2)}{(u^2-v^2)^2} = \frac{4vu^2}{(u^2-v^2)^2}$$

$$\frac{\partial u}{\partial x} = -e^{-x-y}, \quad \frac{\partial u}{\partial y} = -e^{-x-y}$$

$$\frac{\partial v}{\partial x} = y e^{xy}, \quad \frac{\partial v}{\partial y} = x e^{xy}$$

Hence,

$$(*) \quad \frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$= \frac{-4e^{-x-y+2xy}}{(e^{2(-x-y)} - e^{2xy})^2} \cdot (-e^{-x-y})$$

$$+ \frac{4e^{xy+2(-x-y)}}{(e^{2(-x-y)} - e^{2xy})^2} \cdot (y e^{xy})$$

$\dfrac{\partial f}{\partial y}$ is done similarly