Oral Exam Syllabus

Benjamin Balsam

Major Topic Advisor: Professor Alexander Kirillov
Minor Topic Advisor: Professor Oleg Viro

July 3, 2009

Major Topic: Lie Groups and Lie Algebras

1. Rudiments of Lie Theory
   - Basic Definitions and examples
   - Relationship between Lie groups and Lie algebras: exponential map, Lie's theorems, Campbell-Hausdorff formula
   - Universal enveloping algebra, PBW theorem
   - Solvable and nilpotent Lie algebras.
   - Lie's and Engel's theorems.
   - Cartan's criteria for solvability and semisimplicity
2. Representations of Lie groups and Lie algebras
   - Defintion of representation.
   - Irreducible representations; Schur's lemma.
   - Complete reducibility of unitary representations.
   - Compact Lie Groups: Complete reducibility of finite-dimensional representations, matrix coefficients and characters.
3. Representation theory of semisimple Lie algebras
   - Complete reducibility of representations
   - Root decomposition, root systems
   - Weyl group, positive and simple roots
   - Dynkin diagrams and their classification
   - Classification of semisimple Lie algebras
   - Highest weight representations
   - Classification of irreducible-finite dimensional representations
   - Verma modules
   - Representations of $\mathfrak{sl}(n, \mathbb{C})$, Young tableaux
   - Character formula

Minor Topic: Knot Invariants

1. Elementary knot theory

   - Knots, links, Reidemeister moves, oriented/framed knots. ([5],pp.1-8).
   - Kauffman bracket, writhe, Jones polynomial ([5], pp. 8-17)
   - Knot group: Wirtiger presentation
   - Seifert surfaces, genus of a knot. ([4], Chapter 5).
   - Seifert matrices, S-equivalence
   - Alexander polynomial
   - Signature of a knot.

2. Applications

   - Monoidal and braided categories.
   - Equivalence of monoidal categories with strict ones. Maclane's coherence theorem ([2] pp. 288-291).
   - Graphical calculus.
   - Category of ribbon tangles: generators and relations. ([6] pp. 35-40)
   - Reshetikhin-Turaev Theorem: Braided tensor categories yield invariants of links ([6],pp 49-57).
   - Example: $U_q(sl_2)$ as a Hopf algebra: Universal R-matrix, skein relations, relation with Jones polynomial, θ, δ. ([2], pp. 140-142, Chapter 14).

References

1. Fulton, W., Harris, J. *Representation Theory: A First Course*. Graduate Texts in Mathematics, Vol. 129. Springer-Verlag, New York, 1991.

2. Kassel, Christian. *Quantum Groups*. Graduate Texts in Mathematics, Vol. 155. Springer-Verlag, New York, 1995.

3. Kirillov, Alexander, Jr. *An Introduction to Lie Groups and Lie Algebras*. Cambridge University Press, Cambridge, 2008.[1]

4. Murasugi, Kunio. *Knot Theory and Its Applications*. Birkhauser, Boston, 1996.

5. Ohtskui, Tomotada. *Quantum Invariants: A Study of Knots, 3-Manifolds, and Their Sets*. Series on Knots and Everything, Vol. 29. World Scientific, Singapore, 2002.

6. Turaev, V.G. *Quantum Invariants of Knots and 3-Manifolds*. Studies in Mathematics Vol. 18. de Gruyter, New York, 1994.

[1] Primary text for major topic.