Publications and Preprints

with brief abstracts

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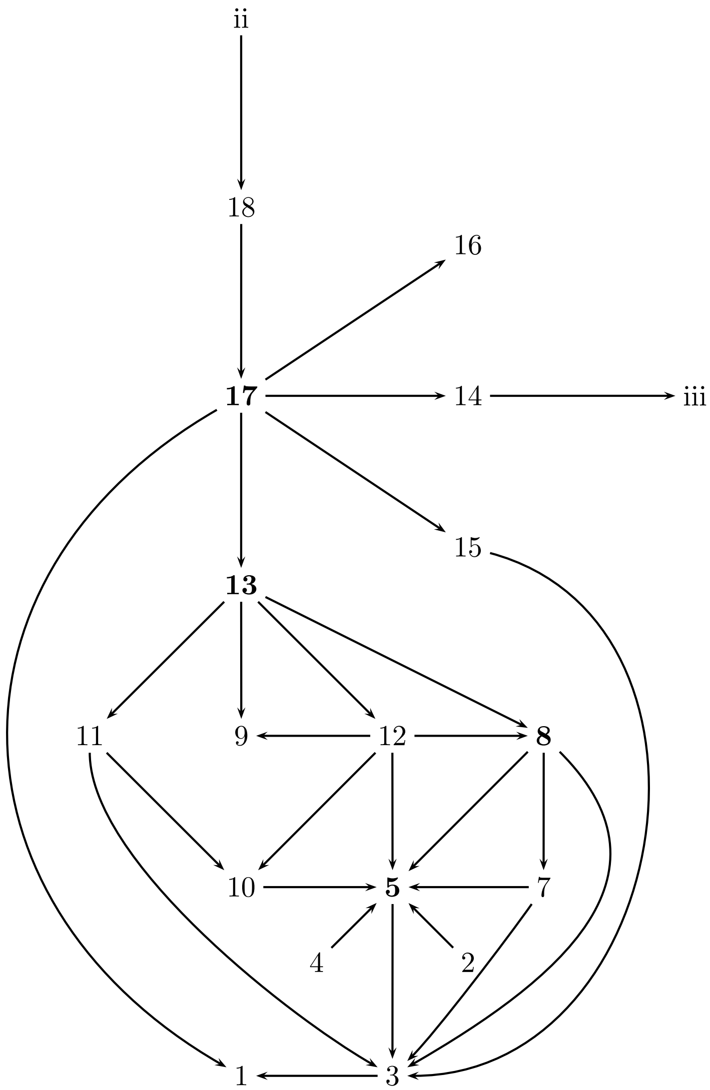
02/11/08

Primary Connections between Papers



17. Analysis of Local Obstructions and Local Excess Intersection Approach
13. A sharp compactness theorem for genus-one pseudoholomorphic maps
8. A smooth compactification of the Hilbert scheme of smooth genus-one curves in $\mathbb{P}^n$
5. Proof of the BCOV prediction for a quintic threefold

Research Papers

1. *Enumerative Geometry of Calabi-Yau 5-Folds*, joint with R. Pandharipande, math/0802.1640, 40 pages, 3 tables, 6 figures; Appendix, 6 pages

   Enumerative geometry for Calabi-Yau 5-folds is defined from Gromov-Witten invariants, following up on Gopakumar-Vafa and Klemm-Pandharipande predictions for CY 3-folds and 4-folds, respectively. A degree scaling for contributions of multiple covers of rational curves to genus 1 GW-invariants is established via analysis of local obstructions motivated by (13), (17), and (18). Integrality tests are carried out for a septic hypersurface in $\mathbb{P}^6$, using closed formulas from (3) and (4), and for the total space of the bundle $3\mathcal{O}(-1)$ over $\mathbb{P}^2$; in the latter case, G. Martin has conjectured an explicit formula for all genus 1 counts.

2. *Some Properties of Hypergeometric Series Associated with Mirror Symmetry*, joint with D. Zagier, math/0710.0889, 14 pages, to appear in BIRS Proceedings, Workshop on Modular Forms and String Duality

   This paper shows that certain hypergeometric series playing a prominent role in mirror symmetry for Calabi-Yau hypersurfaces possess a variety of interesting properties. While these properties appear intriguing in their own right, some of them are also used in (5) to compute the reduced genus-one Gromov-Witten invariants of Calabi-Yau projective hypersurfaces and to verify the 1993 BCOV for a quintic threefold.

3. *Standard vs. Reduced Genus-One Gromov-Witten Invariants*, math/0706.0715, 31 pages, 4 figures

   The difference between the standard genus-one GW-invariants and the reduced genus-one GW-invariants defined in (11) is expressed in terms of genus-zero GW-invariants. The coefficients involve blowups of moduli spaces of genus-one curves constructed in (8) and are computable through the recursive formulas obtained in (7). Along with (8) and (10), this paper makes it possible to compute the standard genus-one GW-invariants of complete intersections. As an application, an explicit closed formula for the standard genus-one GW-invariants of a Calabi-Yau projective hypersurface (of any dimension) is deduced from a closed formula for the reduced invariants obtained in (5). A recent mirror symmetry prediction for a sextic fourfold is thus verified as a special case; the higher-dimensional cases go beyond any predictions.

4. *Genus-Zero Two-Point Hyperplane Integrals in the Gromov-Witten Theory*, math/0705.2397, 32 pages, 4 figures

   This paper explicitly computes two-point analogues of the one-point integrals that are central to the proof of the genus-zero mirror symmetry for a quintic threefold. These integrals are in a sense of arithmetic genus one and are an essential ingredient in the genus-one computation in (5).

5. *The Reduced Genus-One Gromov-Witten Invariants of Calabi-Yau Hypersurfaces*, math/0705.2397, 48 pages, 5 figures

The reduced genus-one Gromov-Witten invariants of Calabi-Yau projective hypersurfaces are computed explicitly by using (10) to reduce the computation to $\overline{\mathfrak{M}}_{1,k}^0(\mathbb{P}^n, d)$ and (8) to apply the classical localization theorem. The 1993 Bershadsky-Cecotti-Ooguri-Vafa (BCOV) prediction for the standard genus-one GW-invariants of a quintic threefold is confirmed as a special case. The summation over the fixed loci is handled by relating the problem to previously carried out genus-zero localization computations and then extracting the non-equivariant part.

6. *Pseudocycles and Integral Homology*, math/0605535, 28 pages, 2 figures, Trans. AMS 360 (2008), 2741-2765

   This paper describes a natural isomorphism between the set of equivalence classes of pseudocycles and the integral homology groups of a smooth compact manifold. This isomorphism has been used widely in symplectic topology. Along the way, observations are made regarding topology of neighborhoods of images of smooth maps and the singular homology of smooth manifolds.

7. *Intersections of Tautological Classes on Blowups of Moduli Spaces of Genus-One Curves*, math/0603357, 4 figures, Mich. Math. 55 (2007), no 3, 535-560

   Two recursions for computing top intersections of tautological classes on blowups of moduli spaces of genus-one curves are obtained in this paper. One of these recursions is analogous to the well-known string equation. As shown in (8) and (10), these numbers are useful for computing genus-one enumerative invariants of projective spaces and Gromov-Witten invariants of complete intersections.

8. *A Desingularization of the Main Component of the Moduli Space of Genus-One Stable Maps into* $\mathbb{P}^n$, joint with R. Vakil, math/0603353, 13 figures, Geom.&Top. 12 (2008) 97-101

   Making use of structural descriptions in (12), (13), and (15), this paper constructs a desingularization of the "main component" $\overline{\mathfrak{M}}_{1,k}^0(\mathbb{P}^n, d)$ of the moduli space of genus-one stable maps into the complex projective space $\mathbb{P}^n$ and a desingularization of a natural cone over $\overline{\mathfrak{M}}_{1,k}^0(\mathbb{P}^n, d)$. Such a desingularization is useful for integrating natural cohomology classes on $\overline{\mathfrak{M}}_{1,k}^0(\mathbb{P}^n, d)$ using localization and thus has applications for computing genus-one Gromov-Witten invariants of complete intersections and genus-one enumerative invariants of projective spaces. As a bonus, we obtain desingularizations of natural sheafs over moduli spaces of genus-one stable maps.

   8b. Overview (independent), 2 figures, ERA AMS, 13 (2007), 53-59

   This short announcement describes the results of the paper from the more classical algebraic geometry perspective of a smooth compactification for the Hilbert scheme of smooth genus-one curves in $\mathbb{P}^n$. Parallels are drawn with the space of complete plane conics, and the highlights of the desingularization construction are illustrated on the space of smooth plane cubics.

   8c. Extended Abstract, Oberwolfach Report 27/2006, 1643-1645

   This extended abstract for Ravi's Oberwolfach talk describes the steps in the desingularization construction and some of the properties of the desingularized moduli space.

9. *On Gromov-Witten Invariants of a Quintic Threefold and a Rigidity Conjecture*, joint with J. Li, math/0406105, 5 figures, Pacific J. Math 233 (2007), no. 2, 417-480.

   This paper rederives a relation for genus-one Gromov-Witten invariants obtained in (10) in the special case of a quintic threefold. The derivation is conditioned on a rigidity conjecture for curves in a quintic threefold and in a sense provides additional evidence for the conjecture. This paper's approach is more direct and geometric than that in (10). It relies heavily on (12) and (13), but not on (11) or the generalization of (12) that constitutes most of (10).

10. *On the Genus-One Gromov-Witten Invariants of Complete Intersections*, joint with J. Li, math/0507104, 45 pages, 4 figures, 1 table

    Most of this paper is devoted to extending the construction of (12) to moduli spaces of perturbed, in a restricted way, J-holomorphic maps. An easy consequence of this extension is a relation between the reduced genus-one Gromov-Witten invariants of complete intersections and the reduced genus-one GW-invariants of the ambient projective space which parallels a well-known relation for the genus-zero invariants. As an application, the standard genus-one GW-invariants of complete intersections can be expressed in terms of the genus-zero and genus-one GW-invariants of projective spaces, solving the genus-one case of a fundamental problem concerning the Gromov-Witten invariants. A relationship for higher-genus invariants is conjectured as well.

11. *Reduced Genus-One Gromov-Witten Invariants*, math/0507103, 48 pages, 5 figures

    In this paper, some of the constructions of (13) are generalized to moduli spaces of perturbed, in a restricted way, J-holomorphic maps. This generalization implies that the main component of the moduli space of J-holomorphic maps carries a virtual fundamental class and defines symplectic invariants. These truly genus-one invariants differ from the standard genus-one Gromov-Witten invariants by a combination of the genus-zero invariants.

12. *On the Structure of Certain Natural Cones over Moduli Spaces of Genus-One Holomorphic Maps*, math/0406104, 3 figures, Adv. Math. 214 (2007), no. 2, 878–933

    It is shown in this paper that certain naturally arising cones over the "main component" of a moduli space of genus-one stable maps have a well-defined euler class, provided the symplectic manifold (X, ω, J) is sufficiently regular. This result is obtained by adapting the power series of (17) for the behavior of derivatives of holomorphic maps under gluing to holomorphic bundle sections.

13. *A Sharp Compactness Theorem for Genus-One Pseudo-Holomorphic Maps*, math/0406103, 78 pages, 7 figures

    This paper gives a sharp version of the stable-map compactness theorem in the genus-one case. If (X, ω, J) is a symplectic manifold satisfying certain regularity conditions, the described "main component" of the moduli space of genus-one stable maps is the closure of the space of maps with smooth domains and carries a rational fundamental class. The compactification theorem is proved by adopting the gluing procedure of (18) and using the power series of (17).

14. *Counting Rational Curves of Arbitrary Shape in Projective Spaces*, math/0210146, 15 figures, 5 tables, Geom.&Top. 9 (2005), 571-697

    This paper describes an algorithm for solving a large class of enumerative problems concerning rational curves in projective spaces by significantly extending the applicability of the topological method and of the power series for derivatives of (17). Its application in each specific case requires only general understanding of the topology of moduli space of holomorphic maps. This paper's approach is demonstrated by enumerating one-component rational curves in $\mathbb{P}^3$ that have a triple point, that have a tacnodal point, and cuspidal curves in $\mathbb{P}^n$.

15. *Enumeration of One-Nodal Rational Curves in Projective Spaces*, math/0204236, 9 figures, 1 table, Topology 43 (2004), no. 4, 793-829

    This paper gives a formula computing the number of genus-one curves with any fixed j-invariant that pass through an appropriate collection of constraints in a complex projective space. Two of the key ingredients are the topological method and the power series for derivatives of (17).

16. *Enumeration of Genus-Three Plane Curves with a Fixed Complex Structure*, math/0203058, 3 tables, J. Algebraic Geom. 14 (2005), no. 1, 35-81

    Using the approach of (17), this paper determines the number of genus-three degree-d plane curves that pass through general $3d-4$ points and have a fixed non-hyperelliptic complex structure on the normalization. An intermediate result is a formula for the number of rational plane curves with a $(3,4)$-cusp.

17. *Enumeration of Genus-Two Curves with a Fixed Complex Structure in $\mathbb{P}^2$ and $\mathbb{P}^3$*, math/0201254, 3 tables, 2 figures, J. Diff. Geom. 65 (2003), no. 3, 341-467

    This paper uses the explicit gluing construction of (18) to obtain power series that describe the behavior of all derivatives of rational holomorphic maps into $\mathbb{P}^n$ under gluing and obstructions to gluing positive-genus maps in some situations. This paper also presents the notion of contribution of a (possibly open) stratum of the zero set of a vector bundle section to the euler class of the bundle and relates this contribution to the number of zeros of an affine bundle map over the closure of the stratum. The main applications are formulas for the number of genus-two curves with a fixed complex structure in $\mathbb{P}^2$ and $\mathbb{P}^3$. Other applications include formulas for the number of rational curves with a cusp in $\mathbb{P}^2$ and $\mathbb{P}^3$.

18. *Enumerative vs. Symplectic Invariants and Obstruction Bundles*, math/0201255, J. Symplectic Geom. 2 (2004), no. 4, 445–543

    This paper presents in detail a gluing construction for pseudoholomorphic maps in symplectic topology, especially in the presence of an obstruction bundle. The explicit nature of this construction leads to two types of power series in (17), which are also used in an essential way in (9)-(16). The main motivation is to try to compare the symplectic and enumerative invariants of algebraic manifolds.

19. *Completion of Katz-Qin-Ruan's Enumeration of Genus-Two Plane Curves*, math/0201216, 3 figures, 1 table, J. Algebraic Geom. 13 (2004), no. 3, 547-561

    In this paper, a formula for the number of genus-two degree-d plane curves that pass through $3d-2$ general points and have a fixed complex structure on the normalization is obtained. This is achieved by completing Katz-Qin-Ruan's work on the same question. The resulting formula agrees with the corresponding formula in (17), which is obtained in a completely different way.

20. *The Exponential Decay Rate of the Lower Bound for the First Eigenvalue of Compact Manifolds*, joint with M. Kalka, E. Mann, and D. Yang, 1 figure, Internat. J. Math. 8 (1997), no. 3, 345-355

    This paper provides the optimal exponential decay rate of the lower bound for the first positive eigenvalue of the Laplacian operator on a compact Riemannian manifold with a negative lower bound on the Ricci curvature and with large diameter.

Miscellaneous

i. *Enumerative Algebraic Geometry via Techniques of Symplectic Topology and Analysis of Local Obstructions*, PhD thesis, 240 pages, 8 tables

   This dissertation contains foundations of the Local Excess Intersection Approach and its applications in Enumerative Geometry, including in finite-dimensional singular and infinite-dimensional Freedholm setting. It incorporates the papers (16)-(19) and includes a detailed introduction.

ii. *Basic Estimates of Riemannian Geometry Used in Gluing Pseudoholomorphic Maps*, 13 pages

    This note collects some basic facts of Riemannian geometry commonly used for gluing pseudoholomorphic maps in symplectic geometry. The dependence of "constants" on the relevant parameters is emphasized.

iii. *Counting Plane Rational Curves: Old and New Approaches*, math/0507105, 32 pages, 2 figures, 2 tables

     These notes are intended as an easy-to-read introduction to a classical and a modern approach in enumerative algebraic geometry. In particular, the numbers n_3 and n_4 of plane rational cubics through eight points and of plane rational quartics through eleven points are determined via the classical approach of counting curves. The computation of the latter number also illustrates the topological method of (17). The arguments used in the computation of the number n_4 extend easily to counting plane curves with two or three nodes, for example. Finally, an inductive formula for the number n_d of plane degree-d rational curves passing through $3d-1$ points is derived via the modern approach of counting stable maps.