# Math53: Ordinary Differential Equations
# Winter 2004

## Homework Assignment 4

**Problem Set 4 is due by 2:15p.m. on Monday, 2/23, in 380Y**

### Problem Set 4:

7.2: 8,14; 7.4: 16,20; 7.5: 14,26; 7.6: 14,28,44; 8.1: 10; 8.2: 13*; 9.1: 6,54;
9.2: 1*,4*,10,24*,26*,30,38*,40*,44; 9.4: 14; 9.5: 8,12,14; 9.8: 6,18,29;
PS4-Problem 30 (see next page)

**Note 1:* In 8.2:13, justify your answers. In 9.2:1,4,24,26,38,40, sketch phase-plane portraits, as in Section 9.3.

*Note 2:* Since this problem set is due on a Monday, I will have office hours 4-6p.m. on Sunday, 2/22.

### Daily Assignments:

| *Date* | *Read* | *Exercises* |
|---|---|---|
| 2/9 M | 7.1-7.4,7.6 | 7.2:8,14; 7.4:16,20; 7.6:14,28,44 |
| 2/10 T | 7.5,9.1 | 7.5:14,26; 9.1:6,54 |
| 2/11 W | 9.5 (pp492-500top) | 9.5:8,12,14 |
| 2/12 R | 8.1,8.2 | 8.1:10; 8.2:13* |
| 2/13 F | 9.2 (pp452-454), 9.3 (pp466-473top) | 9.2:1*,4*,10 |
| | | |
| 2/17 T | 9.2 (pp454-459mid), 9.3 (pp473-479) | 9.2:24*,26*,30 |
| 2/18 W | 9.2 (pp459-463), 9.5 | 9.2:38*,40*,44 |
| 2/19 R | 9.4 | 9.4:14 |
| 2/20 F | 9.8 | 9.8:6,18,29 |
| | | |
| 2/23 M | 8.4,9.6,9.7 | 9.6:7,9; 9.7:17 |

**Note 1:* In 8.2:13, justify your answers. In 9.2:1,4,24,26,38,40, sketch phase-plane portraits, as in Section 9.3.

*Note 2:* Problems 9.6:7,9 and 9.7:17 are not part of Problem Set 4.

## PS4-Problem 30

Recall that we are able to reduce the general *first*-order linear ODE

$$y' + a(t)y = f(t), \qquad y = y(t),$$

to a ready-to-integrate equation $(Py)' = Pf$ by finding an integrating factor $P = P(t)$ such that

$$P' = aP \qquad \Longrightarrow \qquad (Py) = Py' + aPy.$$

Similarly, we can reduce a *second*-order linear ODE *with constant coefficients*

$$y'' + py' + qy = f, \quad y = y(t), \qquad p, q = const,$$

to a first-order linear ODE by multiplying by an integrating factor such that

$$\big(P(y' + ay)\big)' = P(y'' + py' + q),$$

for some function $a = a(t)$. This integrating factor is $P(t) = e^{-\lambda_1 t}$, where $\lambda_1$ is one of the roots of the corresponding characteristic polynomial $\lambda^2 + p\lambda + q = 0$. We cannot adapt this approach to an arbitrary *second*-order linear ODE. Here is why.

(a) Suppose we would like to find smooth nonzero functions $P = P(t)$ and $Q = Q(t)$ such that

$$\big(Q(y' + ay)\big)' = P(y'' + py' + qy), \qquad p = p(t), \; q = q(t), \tag{1}$$

for some smooth function $a = a(t)$ and for every smooth function $y = y(t)$. Show that we must have

$$P = Q, \qquad P' + Pa = Pp, \quad \text{and} \quad (Pa)' = qP.$$

(b) Thus, the functions $P$ and $a$ can be found by finding a nonzero solution to

$$\begin{pmatrix} P \\ (Pa) \end{pmatrix}' = \begin{pmatrix} p & -1 \\ q & 0 \end{pmatrix} \begin{pmatrix} P \\ (Pa) \end{pmatrix} \qquad P = P(t), \; a = a(t).$$

Find a nonzero solution to this ODE if $p$ and $q$ are constant, obtaining an integrating factor for second-order ODEs with constant coefficients. Use it to find $R = R(t)$ such that

$$\big(P(Ry)'\big)' = P(y'' + py' + qy), \qquad p, q = const.$$

(c) Apply the same approach to third-order ODEs. In other words, if $p, q, r = const$, find functions $P = P(t) \neq 0$, $Q = Q(t)$, and $R = R(t)$, such that

$$\Big(P\big(Q(Ry)'\big)'\Big)' = P(y''' + py'' + qy' + ry).$$