Math53: Ordinary Differential Equations
Winter 2004

Homework Assignment 2

Problem Set 2 is due by 2:15p.m. on Monday, 1/26, in 380Y

Problem Set 2:

PS2-Problem 1 (see next page); 4.1: 12,14; 4.2: 4; 4.3: 4,10,14,26; 4.4: 17 (1st part only); 4.5: 2,6,16,18,26,30,32,42.

Note 1: While the statement of Problem 1 looks long, most of it is actually a review.
Note 2: Since this problem set is due on a Monday and there is a number of problems from the preceding Friday, I will have office hours 4-6p.m. on Sunday, 1/25.

Daily Assignments:

Please review complex numbers, pp181-184, before F. 1/15

Date	*Read*	*Exercises*
1/15 R	4.3 (pp181-184)	
1/16 F	4.1 (pp163-167), 4.3	PS2-Problem 1; 4.3: 4,10,14,26
1/20 T	4.4	4.4: 17 (1st part only)
1/21 W	4.1,4.2	4.1: 12,14; 4.2: 4
1/22 R		
1/23 F	4.5	4.5: 2,6,16,18,26,30,32,42
1/26 M	4.6,4.7	4.6: 13

Note: Problem 4.6: 13 is not part of Problem Set 2, but please do it as an exercise.

PS2-Problem 1

As discussed in class, if p and q are constants,

$$\left(e^{(\lambda_2-\lambda_1)t}(e^{-\lambda_2 t}y)'\right)' = e^{-\lambda_1 t}(y''+py'+qy), \tag{1}$$

if λ_1 and λ_2 are the two roots of the characteristic polynomial

$$\lambda^2+p\lambda+q=0 \tag{2}$$

associated to the linear homogeneous second-order ODE

$$y''+py'+qy=0.$$

Thus, every second-order linear ODE with constant coefficients,

$$y''+py'+qy=f(t) \tag{3}$$

can be solved in four steps:
Step 1: find the roots of the associated characteristic polynomial (2);
Step 2: multiply both sides of (3) by $e^{-\lambda_1 t}$;
Step 3: use (1) to compress LHS of the resulting expression and to obtain

$$\left(e^{(\lambda_2-\lambda_1)t}(e^{-\lambda_2 t}y)'\right)' = e^{-\lambda_1 t}f(t); \tag{4}$$

Step 4: solve (4) for y by integrating twice.

This approach mimics the *integrating factor method* for solving linear first-order ODEs, though it works *only* for constant p and q. Its advantage over the methods described in Sections 4.3 and 4.5 of the text is that
(1) it works the same way whether or not λ_1 and λ_2 are distinct;
(2) it works the same way no matter what f looks like.

Use the above second-order integrating factor method to find the *real* (not complex) general solutions of
(a) $y''+4y=4\cos 2t$;
(b) $y''+5y'+4y=t\cdot e^{-t}$.
Compare your answers to (a) and (b) with your answers to 4.5:26 and 4.5:42, after you do them.

Note: When you work on 4.5:26 and 4.5:42, please use the methods requested in the textbook, as opposed to rewriting your solutions for (a) and (b).