# Math53: Ordinary Differential Equations
# Autumn 2004

## Midterm I Practice Tests

*Note:* Midterm I from a previous quarter and its solutions will be posted on the course website,

http://math.stanford.edu/∼azinger/math53,

by Saturday, 10/16.

### Practice Test A

### Problem 1

(a) Find the general solution to the ODE

$$yy' = -2t - 2ty^2.$$

(b) Find an explicit solution, including the interval of existence, to the initial value problem

$$yy' = -2t - 2ty^2, \qquad y(0) = 1.$$

### Problem 2

(a) Show that the ODE

$$1 - y\sin t + (\cos t)y' = 0$$

is exact and solve it for $y=y(t)$, implicitly or explicitly.
(b) Find an explicit solution, including the interval of existence, to the initial value problem

$$1 - y\sin t + (\cos t)y' = 0, \qquad y(0) = 1.$$

### Problem 3

(a) Find the general solution of the ODE $y'' - y = 0$.
(b) Find a solution of the ODE $y'' - y = 2t$.
(c) Find a solution of the ODE $y'' - y = 2e^{-t}$.
(d) Find the general solution of the ODE $y'' - y = 2t - 4e^{-t}$.

### Problem 4

Consider the initial value problem

$$y' = \sqrt{|y|}/|t|, \qquad y(t_0) = y_0.$$

For what initial conditions $(t_0, y_0)$ does the existence and uniqueness theorem for first-order ODEs
(a) guarantee a solution for this IVP?
(b) guarantee a unique solution for this IVP?
Specify all such conditions.

## Practice Test B

### Problem 1

(a) Find the general solution to the ODE

$$ty' = \sin t - 2y.$$

Sketch at least three solution curves.
(b) Find the solution, including the interval of existence, to the initial value problem

$$ty' = \sin t - 2y, \qquad y(\pi/2) = 0.$$

### Problem 2

(a) Show that the substution $y = tv$ reduces the ODE

$$t + y + (y - t)y' = 0 \qquad \text{to} \qquad tv' = \frac{1 + v^2}{1 - v}.$$

(b) Find the general solution to the latter ODE.
(c) Find the solution, explicitly or implicitly, to the initial value problem

$$t + y + (y - t)y' = 0, \qquad y(1) = 0.$$

### Problem 3

(a) Find the general solution of the ODE $y'' + 4y' = 0$.
(b) Find a solution of the ODE $y'' + 4y' = 20\cos 2t$.
(c) Find the solution to the initial value problem

$$y'' + 4y' = 16\cos 2t, \quad y(0) = 1, \quad y'(0) = 1.$$

### Problem 4

(a) Sketch the graph of the function

$$f(y) = (y + 3)^2(y + 1)(y - 3).$$

(b) Find and sketch the equilibrium solutions of the ODE

$$y' = (y + 3)^2(y + 1)(y - 3).$$

(c) On the same plot, sketch at least one solution curve of this ODE in each region of the $ty$-plane cut out by the graphs of the equilibrium solutions. Indicate the asymptotic behavior. Determine whether each of the equilibrium solutions is asymptotically stable or unstable. Draw the phase line.

*Remarks:* Problems A1, A2, A3, B1, B2, B3 are similar to 2.2:14, 2.6:10, 4.5:32, 2.4:13, 2.6:36, and 4.5:26; see PS1, PS2, and PS3 solutions. Problems A3 and B3 also make use of 4.5:30. For Problems A4 and B4, see *Unit 1 Summary*. The two practice tests are meant to indicate some problems that might appear on the midterm and its length, and *not* that the midterm problems would come from the problem sets.