# MAT 566: Differential Topology
# Fall 2006

## Problem Set 7
**Due on Thursday, 12/14, by 5pm in Math 3-117**

Please do either Problem vii or *two* of the following problems: viii, ix, 19-A, 19-B.

Problem (vii): For this problem, assume Hurewicz Theorem (standard and torsion versions), homotopy l.e.s. for fibration, and that $\pi_i(S^{2n-1})$ is finite unless $i = 2n-1$. Let

$$W^{2n-1} = \{(v, w) \in \mathbb{R}^{n+1} : |v|, |w| = 1,\ v \perp w\}.$$

(a) If $n$ is odd, to what simpler space is $W^{2n-1}$ diffeomorphic to? What is its homology?
(b) Suppose $n$ is even. Determine the cohomology and homology of $W^{2n-1}$, at least mod torsion. Determine $\pi_i(W^{2n-1})$ for $i \leq 2n-1$, at least mod torsion. Determine $\pi_i(S^n)$ for $i \leq 2n-1$, at least mod torsion.

Problem (viii): Suppose $n \geq 2$, $\pi_i(X) = 0$ and $\pi_i(Y) = 0$ for all $i < n$, and the Hurewicz homomorphisms

$$h_i : \pi_i(X) \longrightarrow H_i(X; \mathbb{Z}) \qquad \text{and} \qquad \pi_i(Y) \longrightarrow H_i(Y; \mathbb{Z})$$

are isomorphisms mod torsion for all $i < 2n-1$. Show that

$$h_i : \pi_i(X \vee Y) \longrightarrow H_i(X \vee Y; \mathbb{Z})$$

is also an isomorphism mod torsion for all $i < 2n-1$.
*Note:* This is part of the proof of Theorem 18.3, but the book's proof is rather sketchy.

Problem (ix): Show that $\pi_n(S^1 \vee S^n)$ is not finitely generated for all $n \geq 2$.