MAT 541: Algebraic Topology
Suggested Problems for Week 13

You may hand in solutions to at *most* 2 problems every 2 weeks and no later than 2 weeks after the necessary material for them is covered in class.

From Munkres: 61.1, 61.3, 52.3, 52.4, 53.3

Problem V

Give $\mathbb{Z}$ the discrete topology. The module $S_0(\mathbb{Z}\times\mathbb{Z})$ is free with basis

$$\sigma_{ij}\colon \Delta^0 \longrightarrow \mathbb{Z}\times\mathbb{Z}, \quad \sigma_{ij}\big(\Delta^0\big)=\big\{(i,j)\big\}, \qquad i,j\in\mathbb{Z}.$$

Define

$$\delta \in S^0(\mathbb{Z}\times\mathbb{Z}) \qquad \text{by} \qquad \delta\big(\sigma_{ij}\big) = \begin{cases} 1, & \text{if } i=j; \\ 0, & \text{if } i\neq j. \end{cases}$$

(a) Show that δ is a cocycle.

(b) Show that δ is not in the image of $\times\colon H^*(\mathbb{Z})\otimes H^*(\mathbb{Z}) \longrightarrow H^*(\mathbb{Z}\times\mathbb{Z})$.

(c) Let $S\subset\mathbb{Z}$ be a finite subset. Write $\delta|_{\mathbb{Z}\times S}$ explicitly as an element in the image of

$$\times\colon H^*(\mathbb{Z})\otimes H^*(S) \longrightarrow H^*(\mathbb{Z}\times S).$$

This problem illustrates the necessity of the assumption in the Kenneth Theorem for H^* that H_* of one of the factors be finitely generated in each dimension.