

MAT 531: Topology & Geometry, II

Spring 2006

Overview

- Smooth manifolds, tangent vectors, differentials, immersions, etc.
 - PS1 #1-4c; PS2 #1-3
- Vector Bundles
 - PS1 #4d-6; PS2 #4; PS4 #4; PS6 #4a
- Differentials, Inverse FT, Slice Lemma, Implicit FT (I&II)
 - PS3 #1-3; MT #2
- Flows of Vector Fields, Lie Bracket, Lie Derivative
 - PS3 #4-6; PS4 #1,2; PS5 #3,4; PS6 #7a; MT #1
- The Differential $d: E^p(M) \longrightarrow E^{p+1}(M)$, Frobenius Theorem (I&II), Strong Slice Statement
 - PS4 #4; PS5 #1,2,5; MT #3
- de Rham Cochain Complex, Poincaré Lemma, Stokes' Theorem (I&II), and Group Actions
 - PS6 #1,2,6,8; PS6 #5b; PS7 #5; MT #4; PS10 #3
- Orientability of Manifolds and Vector Bundles, Relations with Topology and Covering Maps
 - PS4 #3; PS6 #3-6,7bc; MT #5
- Singular Chain Complex, Hurewicz Theorem
 - PS7 #1

- (Co)Chain Complexes and (Co)homology, Duals, Coefficient Changes
 - Mayer-Vietoris for de Rham Cohomology and Singular Homology: PS7 #2-4
 - Sheafs and Čech Cohomology: PS7 #6,7; PS8 #2,3
 - Cohomology from Fine Resolutions: de Rham Theorem
 - Compactly Supported Cohomology

- Geometric Analysis
 - Differential Operators, Symbol, Elliptic Operators
 - Sobolev Lemma, Rellich Lemma, Fundamental Inequality: PS 10, #4,5

- Hodge Theory
 - Laplacian: PS3 #3; PS10 #1,2
 - Hodge Decomposition Theorem
 - Poincare Duality, Finite-Dimensionality of de Rham Cohomology
 - Kunneth Formula: PS10 #5

- de Rham Cohomology in special cases
 - $H_{\text{deR}}^0(M)$; $H_{\text{deR}}^{\text{top}}(M)$ (M orientable/non-orientable, compact/non-compact): PS10 #3
 - $H_{\text{deR}}^*(\mathbb{R}^n)$, $H_{\text{deR}}^*(S^n)$: PS7 #3