MAT 531: Topology&Geometry, II
Spring 2010

Problem Set 2
Due on Thursday, 2/11, in class

Give concise, but complete, solutions. The entire problem set should not require more than a few pages.

1. Chapter 1, #10 (p51)
2. Chapter 1, #5 (p50)
3. Show that the tangent bundle TM of a smooth n-manifold M is a real vector bundle of rank n over M. What is its transition data?
4. Show that the tangent bundle TS^1 of S^1, defined as in 1.25 (p19), is isomorphic to the trivial real line bundle over S^1.
   Hint: Use Lemma 2.3 in *Notes on Vector Bundles*.
5. Show that the tautological line bundle $\gamma_n \longrightarrow \mathbb{C}P^n$ is indeed a complex line bundle (describe its trivializations). What is its transition data? Why is it non-trivial for $n \geq 1$? (not isomorphic to $\mathbb{C}P^n \times \mathbb{C} \longrightarrow \mathbb{C}P^n$ as line bundle over $\mathbb{C}P^n$)
   Hint: See proof of Lemma 2.2 in *Notes on Vector Bundles*.
6. Suppose $k<n$. Show that the map
$$\iota\colon \mathbb{C}P^k \longrightarrow \mathbb{C}P^n, \qquad [X_0,\ldots,X_k] \longrightarrow [X_0,\ldots,X_k,\underbrace{0,\ldots,0}_{n-k}],$$
   is a complex embedding (i.e. a smooth embedding that induces holomorphic maps between the charts that determine the complex structures on $\mathbb{C}P^k$ and $\mathbb{C}P^n$). Show that the normal bundle to this immersion, $\mathcal{N}_\iota$, is isomorphic to
$$(n-k)\gamma_k^* \equiv \underbrace{\gamma_k^* \oplus \ldots \oplus \gamma_k^*}_{n-k},$$
   where $\gamma_k \longrightarrow \mathbb{C}P^k$ is the tautological line bundle (isomorphic as complex line bundles).
   Hint: There are a number of ways of doing this, including:
   (i) construct an isomorphism between the two vector bundles;
   (ii) use Problems 3 from PS1 and 3,5 above to determine transition data for $\mathcal{N}_\iota$ and $(n–k)\gamma_k^*$;
   (iii) show that there exists a diffeomorphism between $(n-k)\gamma_k^*$ and a neighborhood of $\iota(\mathbb{C}P^k)$ in $\mathbb{C}P^n$ and that this implies that $\mathcal{N}_\iota = (n-k)\gamma_k^*$.
7. Chapter 1, #7 (p50)

Final Exam

The current schedule for final exams is

Tues., 5/11, MAT 535; Wed., 5/12, MAT 550

Th., 5/13, 11:15-1:45 MAT 542 and 5:15-7:45 MAT 531

Mon., 5/17, Calculus

Given this schedule, I'd like to suggest moving the 531 final to some other day, such as Sat-Mon 5/8-10 or Fri-Sun 5/14-16 (a later day is not possible because the semester grades are due within 48 hours, excluding weekends, of the scheduled final exam). The last class is on 5/6 and will likely be just review. I will have office hours the day before the final (whether this falls on a weekday or on a weekend day). If you are registered in this course, on 2/11 please turn in a separate sheet of paper with your thoughts on this, i.e. that you do not want to reschedule the final at all (or to a specific day) and/or list your preferences in order for the day and starting time of the exam (say, 10, 11, 12, 1, 2, and 3). It might be simplest if all of you could agree on the day and time of the final exam before 2/11.