Topology MAT 530 Midterm

Part 1: Theoretical questions. (Provide complete proofs)

   1. Prove that the unit segment $[0, 1] \subset \mathbb{R}$ is compact.

   2. Prove that any compact Hausdorff space is regular.

Part 2: Quiz. (Give answers without proofs)

   1. Is every metric space first-countable?

   2. Is every metric space second-countable?

   3. Is every path connected space connected?

   4. Is every connected space path connected?

   5. What are the topological spaces such that all functions on them are continuous?

   6. Is it true that the product of metrizable spaces is always metrizable?

   7. Is every subspace of $\mathbb{R}^n$ regular?

   8. Is every subspace of $\mathbb{R}^n$ normal?

   9. Is every subspace of $\mathbb{R}^n$ first-countable?

   10. Is every subspace of $\mathbb{R}^n$ second-countable?

   11. Is every closed subspace of $\mathbb{R}^n$ compact?

   12. Is every open subspace of $\mathbb{R}^n$ connected?

   13. Is every open subspace of $\mathbb{R}^n$ path connected?

   14. Remove 3 distinct points from a circle. How many connected components does the resulting space have?

   15. State the Baire theorem.

   16. Define uniformly continuous families of functions.

   17. State the Ascoli theorem.