Topology MAT 530 Homework 7

1. Show that the surface $K^2 \# \mathbb{R}P^2$ is homeomorphic to the real projective plane with a handle. (A proof may be not quite rigorous, but it must be convincing).

2. The fundamental group of the sphere with 2 handles is not commutative.

3. Prove that the space of all affine lines in the plane is homeomorphic to the Möbius strip.

4. Prove that there is no covering of the torus by the sphere with 2 handles.

5. Prove algebraically that the group generated by elements a and b subject to the relation $ab^{-1}ab = 1$ is isomorphic to the group generated by elements c and d subject to the relation $c^2 = d^2$. (We proved this topologically by showing that both groups are isomorphic to the fundamental group of the Klein bottle).