MAT 324: Real Analysis, Fall 2017
First-Day Quiz (40 mins)

Name:

1 (1pt) $10^{\ln 6} - 6^{\ln 10}$ equals

(A) 0 (B) 1 (C) -1 (D) $4^{-\ln 4}$ (E) $\dfrac{1}{256}$

Answer only

2 (1pt) The graph of the function $y=(x-2)^3+3$ is obtained by shifting the graph of $y=x^3$

(A) 3 units up and 2 units left (B) 3 units up and 2 units right
(C) 3 units down and 2 units left (D) 3 units down and 2 units right
(E) 3 units right and 2 units down

Answer only

3 (3pts) $\displaystyle\lim_{x\longrightarrow 0}\frac{\tan x}{x}$ equals

(A) 0 (B) $\dfrac{1}{4}$ (C) $\dfrac{1}{2}$ (D) 1 (E) ∞

Justify your answer

4 (5pts) Which of the following statements is true?

(A) $\displaystyle\int_2^{12} x^{-3/2}\mathrm{d}x < \sum_{n=3}^{n=12} n^{-3/2} < \sum_{n=2}^{n=11} n^{-3/2}$

(B) $\displaystyle\sum_{n=3}^{n=12} n^{-3/2} < \int_2^{12} x^{-3/2}\mathrm{d}x < \sum_{n=2}^{n=11} n^{-3/2}$

(C) $\displaystyle\sum_{n=3}^{n=11} n^{-3/2} < \int_2^{12} x^{-3/2}\mathrm{d}x < \sum_{n=3}^{n=12} n^{-3/2}$

(D) $\displaystyle\sum_{n=2}^{n=12} n^{-3/2} < \sum_{n=1}^{n=11} n^{-3/2} < \int_2^{12} x^{-3/2}\mathrm{d}x$

(E) $\displaystyle\sum_{n=2}^{n=12} n^{-3/2} < \int_2^{12} x^{-3/2}\mathrm{d}x < \sum_{n=1}^{n=11} n^{-3/2}$

Justify your answer

5-10 (1pt each) Let $f\colon X \longrightarrow Y$ be a map (defined on all of X), $X_1, X_2 \subset X$, and $Y_1, Y_2 \subset Y$. Which of the following statements are true? *Answer only*

5 (A) $f(X_1 \cup X_2) \subset f(X_1) \cup f(X_2)$ (B) $f(X_1 \cup X_2) \supset f(X_1) \cup f(X_2)$
(C) both (A) and (B) (D) neither (A) nor (B)

6 (A) $f(X_1 \cap X_2) \subset f(X_1) \cap f(X_2)$ (B) $f(X_1 \cap X_2) \supset f(X_1) \cap f(X_2)$
(C) both (A) and (B) (D) neither (A) nor (B)

7 (A) $f(X \setminus X_1) \subset Y \setminus f(X_1)$ (B) $f(X \setminus X_1) \supset Y \setminus f(X_1)$
(C) both (A) and (B) (D) neither (A) nor (B)

8 (A) $f^{-1}(Y_1 \cup Y_2) \subset f^{-1}(Y_1) \cup f^{-1}(Y_2)$ (B) $f^{-1}(Y_1 \cup Y_2) \supset f^{-1}(Y_1) \cup f^{-1}(Y_2)$
(C) both (A) and (B) (D) neither (A) nor (B)

9 (A) $f^{-1}(Y_1 \cap Y_2) \subset f^{-1}(Y_1) \cap f^{-1}(Y_2)$ (B) $f^{-1}(Y_1 \cap Y_2) \supset f^{-1}(Y_1) \cap f^{-1}(Y_2)$
(C) both (A) and (B) (D) neither (A) nor (B)

10 (A) $f^{-1}(Y \setminus Y_1) \subset X \setminus f^{-1}(Y_1)$ (B) $f^{-1}(Y \setminus Y_1) \supset X \setminus f^{-1}(Y_1)$
(C) both (A) and (B) (D) neither (A) nor (B)

11-13 (3pts each) Show work below.

11 Let $a_1, a_2, \ldots$ be a sequence of 0's and 1's. Show that the series $\displaystyle\sum_{k=1}^{\infty} \frac{a_k}{3^k}$ converges to some number $a \in [0,1]$.

12 If $a'_1, a'_2, \ldots$ is a different sequence of 0's and 1's (i.e. $a_k \neq a'_k$ for at least one $k \in \mathbb{Z}^+$), show that

$$\sum_{k=1}^{\infty} \frac{a_k}{3^k} \neq \sum_{k=1}^{\infty} \frac{a'_k}{3^k}$$

13 Explain why the set $\mathbb{Q}$ of rational numbers is countable, but the set $\mathbb{R}$ of real numbers is uncountable.

14 (3pts) Let Ω be a set consisting of 10 elements. How many distinct subsets does Ω contain? *Justify your answer*

15 (7pts) Show that the function

$$1_{\mathbb{Q}} : [0,1] \longrightarrow \mathbb{R}, \qquad 1_{\mathbb{Q}}(x) = \begin{cases} 1, & \text{if } x \in [0,1] \cap \mathbb{Q}; \\ 0, & \text{if } x \in [0,1] \setminus \mathbb{Q}; \end{cases}$$

is not Riemann-integrable.