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If  $A = A_1 \times A_2$ , then the function  $\omega_2 \mapsto P(A_{\omega_2})$  is a step function:

$$P(A_{\omega_2}) = \begin{cases} P(A_1) & \text{if } \omega_2 \in A_2 \\ 0 & \text{if } \omega_2 \notin A_2 \end{cases}$$

and hence we have

$$P(A) = P_1(A_1)P_2(A_2) = \int_{\Omega_2} P(A_{\omega_2}) dP_2(\omega_2).$$

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(6.3)

$$\int_{\Omega_1} P_1(A_{\omega_2}) dP_2(\omega_2) = \int_{\Omega_2} P_2(A_{\omega_1}) dP_1(\omega_1). \quad (1)$$

replaced by

$$\int_{\Omega_2} P_1(A_{\omega_2}) dP_2(\omega_2) = \int_{\Omega_1} P_2(A_{\omega_1}) dP_1(\omega_1). \quad (2)$$

65, Theorem 3.12

### Theorem 0.1

If  $f : E \rightarrow \mathbb{R}$  is measurable,  $E \in \mathcal{M}$ ,  $g : E \rightarrow \mathbb{R}$  is such that the set  $\{x : f(x) = g(x)\}$  is null, then  $g$  is measurable.

replaced by

### Theorem 0.2

If  $f : E \rightarrow \mathbb{R}$  is measurable,  $E \in \mathcal{M}$ ,  $g : E \rightarrow \mathbb{R}$  is such that the set  $\{x : f(x) \neq g(x)\}$  is null, then  $g$  is measurable.

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But the  $(E_n)$  are disjoint: to see this, suppose that  $z \in E_m \cap E_n$  for some  $m \neq n$ .

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### Proposition 0.3

The completion of  $\mathcal{G}$  is of the form  $\{G \cup N : G \in \mathcal{F}, N \subset F \in \mathcal{F} \text{ with } \mu(F) = 0\}$ .

replaced by

### Proposition 0.4

The completion of  $\mathcal{G}$  is of the form  $\{G \cup N : G \in \mathcal{G}, N \subset F \in \mathcal{F} \text{ with } \mu(F) = 0\}$ .

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Fix  $m < n$  and define a  $\sigma$ -field  $\mathcal{F}_m = \{A : \omega, \omega' \in A \implies \omega_1 = \omega'_1, \omega_2 = \omega'_2, \dots, \omega_m = \omega'_m\}$ . So all paths from a particular set  $A$  in this  $\sigma$ -field have identical initial segments while the remaining coordinates are arbitrary. Note that

replaced by

Fix  $m < N$   $\mathcal{F}_m$  to be the  $\sigma$ -field generated by the following family of sets  $\{A : \omega, \omega' \in A \implies \omega_1 = \omega'_1, \omega_2 = \omega'_2, \dots, \omega_m = \omega'_m\}$ . So all paths from a particular set  $A$  in this  $\sigma$ -field have identical initial segments while the remaining coordinates are arbitrary. Note that

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$$f^{-1}(I) = \{x \in \mathbb{R} : f(x) \in I\} \in \mathcal{M}.$$

replaced by

$$f^{-1}(I) = \{x \in E : f(x) \in I\} \in \mathcal{M}.$$

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$$X^{-1}(\mathcal{B}) = \{S \subset \mathcal{F} : S = X^{-1}(B) \text{ for some } B \in \mathcal{B}\}$$

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The simplest possible case is where  $X$  is constant,  $X \equiv a$ . The  $X^{-1}(B)$  is either  $\Omega$  or  $\emptyset$ , depending on whether  $a \in B$  or not and the  $\sigma$ -field generated is trivial:  $\mathcal{F} = \{\emptyset, \Omega\}$ .

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### *Exercise 0.1*

Find the integral of  $\varphi$  over  $E$  where

(a)  $\varphi(x) = \text{Int}(x)$ ,  $E = [0, 10]$ ,

(b)  $\varphi(x) = \text{Int}(x^2)$ ,  $E = [0, 2]$ ,

(c)  $\varphi(x) = \text{Int}(\sin x)$ ,  $E = [0, 2\pi]$

and  $\text{Int}$  denotes the integer part of a real number. (Note that many texts use the symbol  $[x]$  to denote  $\text{Int}(x)$ . We prefer to use  $\text{Int}$  for increased clarity.)

replaced by

### *Exercise 0.2*

Find the integral of  $\varphi$  over  $E$  where

(a)  $\varphi(x) = \text{Int}(x)$ ,  $E = [0, 10]$ ,

(b)  $\varphi(x) = \text{Int}(x^2)$ ,  $E = [0, 2]$ ,

(c)  $\varphi(x) = \text{Round}(\sin x)$ ,  $E = [0, 2\pi]$

where  $\text{Int}(x)$  denotes the integer part of  $x$  and  $\text{Round}(x)$  is the integer nearest to  $x$ ,  $\text{Round}(0.5)$  being zero. (Note that many texts use the symbol  $[x]$  to denote  $\text{Int}(x)$ . We prefer to use  $\text{Int}$  for increased clarity.)

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**Step 4.** Finally, let  $f, g$  be arbitrary integrable functions. Since

$$\int_E |f + g| \, dm \leq \int_E (|f| + |g|) \, dm,$$

we can use Step 2 to deduce that the left-hand side is finite.

replaced by

**Step 4.** Finally, let  $f, g$  be arbitrary integrable functions. Since

$$\int_E |f + g| \, dm \leq \int_E (|f| + |g|) \, dm,$$

we can use Step 3 to deduce that the left-hand side is finite.