MAT 324: Real Analysis, Fall 2017

Comments on and Additional Corrections to the Textbook

November 29, 2017

The corrections listed below are in addition to those listed in the official erratum provided by the authors.

p ix, line -9: *not* should not be there
p9, 2nd displayed equation: lower limit should be a
p13, line -8: *and* $\longrightarrow$ *or*

p19, Exercise 2.2: the wording is slightly off. For example, $x=1/3$ lies in the Cantor set C even though $a_1=1$. On the other hand,

$$\frac{1}{3}=\sum_{k=2}^{\infty}\frac{2}{3^k}=.0222\ldots \quad \text{base 3.}$$

The statement of the exercise becomes correct if every $x\in(0,1)$ is written as an *infinite* expansion base 3. This is always possible, since the last nonzero digit in a finite expansion can be lowered by 1 and then followed by the infinite tail of all 2's. This statement also becomes correct with the word *iff* on the last line replaced by *if*. This would be sufficient to show that C is uncountable. In either case, one needs to show that the infinite expansions involving distinct sequences of a_k sum up to different numbers.

p21, before Theorem 2.4: there should be a statement here analogous to Exercise 2.1. This would remove the need for repeating the same argument of passing to open subsets in the proofs of Theorems 2.6 and 2.17

pp22-24, proof of Theorem 2.6, Step 2: this also needs to be established for infinite intervals. The desired conclusion follows from Proposition 2.5 and the conclusion for the finite intervals. For example,

$$m^*\big([a,\infty)\big)\geq m^*\big([a,a+n]\big)\geq \ell\big([a,a+n]\big)=n \qquad \forall\ n\in\mathbb{Z}^+.$$

Thus, $m^*([a,\infty))=\infty=\ell([a,\infty))$.

p27, top: the *continuity property* as stated is not reasonable and is not satisfied by the measure m on $\mathbb{R}$. For example, the sets $E_n\equiv[n,n+1)$ are pairwise disjoint, but the lengths of

$$B_n\equiv\bigcup_{k=1}^{\infty}E_k-\bigcup_{k=1}^{n}E_k=[n+1,\infty)$$

do not *decrease to* 0 *as* $n\longrightarrow\infty$. This issue is related to the issue with Theorem 2.21 on p39 discussed below.

p28, (ii), first displayed equation: first inequality is not used below
p36, Proposition 2.16: $m(A\Delta B)\ \longrightarrow m^*(A\Delta B)$; $m(B)=m(A)$ would be more logical

p36, Theorem 2.17: It would be more natural to re-write the corresponding statements in the book as

$$m^*(A) = m^*(\bigcap_n O_n), \qquad \sum_{n=1}^{\infty} l(I_n) \le m^*(A) + \frac{\varepsilon}{2}\,.$$

The last inequality above is used directly in the last inequality on p36. It would have also been natural to conclude the statement of Theorem 2.17(ii) in the same way as the statement of Theorem 2.17(i), but with $\varepsilon = 0$; this would have greatly simplified the proof of Theorem 2.28.

p38, (ii): a separate argument is needed if $m(A_n) = \infty$ for all n

p39, Theorem 2.21: (B_n) *decreases to* $\emptyset$ presumably means

$$B_1 \supset B_2 \supset \ldots \qquad \text{and} \qquad \bigcap_{n=1}^{\infty} B_n = \emptyset\,.$$

However, one also needs to assume that $m(B_k) < \infty$ for some $k \in \mathbb{Z}^+$, as is used in the proof. Otherwise, (ii) is not true; the sets $B_n \equiv (n, \infty)$ provide a counter-example. With this interpretation, (ii) is a special case of Theorem 2.19(ii). Since (i) is a special case of Theorem 2.11(iii), Theorem 2.21 seems to be a completely pointless statement. According to the sentence preceding this theorem, the two properties *characterize countably additive set functions* (presumably on σ-fields). However, this is not the case once the statement of (ii) is corrected as above. For example, let

$$\Omega = \mathbb{Z}^+, \qquad \mathcal{F} = 2^{\Omega}, \qquad \mu(S) = \begin{cases} \sum\limits_{i \in S} 2^{-i}, & \text{if } |S| < \infty; \\ \infty, & \text{if } |S| = \infty. \end{cases}$$

The collection $\mathcal{F}$ of all subsets S of Ω is a σ-field. The $[0, \infty]$-valued function μ on $\mathcal{F}$ satisfies (i) as stated and (ii) if $\mu(B_k) < \infty$ for some $k \in \mathbb{Z}^+$. However, μ is not countably additive.

There are actually two notions of continuity for a set function $\mu\colon \mathcal{F} \longrightarrow [0, \infty]$ on a σ-field $\mathcal{F}$ on a set Ω. Such a function μ is called *continuous from below* if

$$E_1, E_2, \ldots \in \mathcal{F}, \quad E_1 \subset E_2 \subset \ldots \qquad \Longrightarrow \qquad \mu\Big(\bigcup_{n=1}^{\infty} E_n\Big) = \lim_{n \longrightarrow \infty} \mu(E_n).$$

It is called *continuous from above* if

$$E_1, E_2, \ldots \in \mathcal{F}, \quad E_1 \supset E_2 \supset \ldots \qquad \Longrightarrow \qquad \mu\Big(\bigcap_{n=1}^{\infty} E_n\Big) = \lim_{n \longrightarrow \infty} \mu(E_n).$$

A finitely additive set function μ is countably additive if and only if it is *continuous from below.* If $\mu(\Omega) < \infty$, then a finitely additive set function μ is countably additive if and only if it is *continuous from above.* The only if parts of these statements have been proved in class (Theorem 2.19). The converses are not too difficult and involve essentially reversing the proof of Theorem 2.19.

At the top of p27 and in Theorem 2.21(ii) on p39, *continuity* means *continuity from above.* However, the second equivalence stated above does not apply to the set function $\mu = m$ on the σ-field $\mathcal{F} = \mathcal{M}$ of measurable subsets of $\Omega = \mathbb{R}$ because $\mu(\Omega) = m(\mathbb{R}) = \infty$. The authors are perhaps thinking of probability measure for which $\mu(\Omega) = 1$ and so the second equivalence stated above applies.

p42, line -2: *m-null* $\longrightarrow$ *m^*-null*
p43, proof of Theorem 2.28: the wording is nominally correct, but the argument is overly complicated. If $N \subset B$ for some $B \in \mathcal{B}$ with $m^*(B) = 0$, then N is null and thus $N \in \mathcal{B}$. The first paragraph in the book's proof essentially repeats the proof of Theorem 2.10(i). The last paragraph would not have been necessary if the statement of Theorem 2.17(ii) had concluded in the same way as the statement of Theorem 2.17(i), but with $\varepsilon = 0$.

p44, Theorem 2.29: the wording is nominally correct, but the theorem itself should appear immediately after Theorem 2.17. This is simply about taking complements and their relation with open/closed and inclusions/containments.

p48, line 5 after Exercise 2.10: *independentif* $\longrightarrow$ *independent if*

p50 bottom, p51 top: this part is completely messed up. As written in the textbook, there is no condition imposed on a subset $A \subset \Omega$ to be in $\mathcal{F}_0$. Thus, $\mathcal{F}_0 = 2^\Omega$ is the set of all subsets of Ω and so contains 2^{2^N} elements. According to the official erratum, $\mathcal{F}_m$ is the σ-field generated by the collection of subsets $A \subset \Omega$ consisting of the sequences $\omega \equiv (\omega_1, \ldots, \omega_N)$ so that $\omega_1 = \omega'_1, \ldots, \omega_m = \omega'_m$ for any two elements $\omega, \omega' \in A$. This imposes no condition on the one-element subsets $A = \{\omega\}$ of Ω. Thus, $\mathcal{F}_m$ contains all one-element subsets of ω. Since $\mathcal{F}_m$ is a σ-field, it is closed under countable unions and intersections of its subsets and thus contains all subsets of Ω, i.e. $\mathcal{F}_m = 2^\Omega$ and contains 2^{2^N} elements as in the $m = 0$ case.

It appears that the authors meant that $\mathcal{F}_m$ is the σ-field generated by the collection of *maximal* subsets $A \subset \Omega$ consisting of the sequences $\omega \equiv (\omega_1, \ldots, \omega_N)$ so that $\omega_1 = \omega'_1, \ldots, \omega_m = \omega'_m$ for any two elements $\omega, \omega' \in A$. The *maximal* condition means that for each generator A and for every sequence $(\omega'_1, \ldots, \omega'_N)$ not in A, there is a sequence $(\omega_1, \ldots, \omega_N)$ in A such that $\omega_i \neq \omega'_i$ for some $i = 1, \ldots, m$. Each generator A then corresponds to a path $\omega = (\omega_1, \ldots, \omega_m)$ of $m \leq N$ steps,

$$A_\omega = \left\{ \omega' \equiv (\omega'_1, \ldots, \omega'_N) \in \Omega : \omega'_1 = \omega_1, \ldots, \omega'_m = \omega_m \right\}.$$

There are then 2^m such generators. Their union is all of Ω (every sequence $\omega' \in \Omega$ lies in some A_ω). The collection of subsets of Ω obtained by taking the unions of the generators is closed under taking the complement (if $A \subset \Omega$ is in this collection, then so is $A^c = \Omega \setminus A$). Thus, $\mathcal{F}_m$ is the collection of subsets of Ω obtained by taking the unions of the generators, including the union of the empty subset of the set of the generators (this union is the empty set).

The σ-field $\mathcal{F}_0$ is generated by Ω alone. It thus consists of the empty set $\emptyset$ (the union of the empty subset of the set of the generators) and of the entire space Ω (the union of the entire set of the generators). The σ-field $\mathcal{F}_1$ is generated by

$$A_1 \equiv \{\omega' \equiv (\omega'_1, \dots, \omega'_N) \in \Omega : \omega'_1 = 1\} \quad \text{and} \quad A_1^c = A_0 \equiv \{\omega' \equiv (\omega'_1, \dots, \omega'_N) \in \Omega : \omega'_1 = 0\}.$$

This σ-field thus consists of $\emptyset$, A_0, A_1, and $\Omega = A_0 \cup A_1$. The σ-field $\mathcal{F}_1$ is generated by 2^2 elements of Ω:

$$A_{00} \equiv \{\omega' \equiv (\omega'_1, \dots, \omega'_N) \in \Omega : \omega'_1 = 0,\ \omega'_2 = 0\}, \quad A_{01} \equiv \{\omega' \equiv (\omega'_1, \dots, \omega'_N) \in \Omega : \omega'_1 = 0,\ \omega'_2 = 1\},$$
$$A_{10} \equiv \{\omega' \equiv (\omega'_1, \dots, \omega'_N) \in \Omega : \omega'_1 = 1,\ \omega'_2 = 0\}, \quad A_{11} \equiv \{\omega' \equiv (\omega'_1, \dots, \omega'_N) \in \Omega : \omega'_1 = 1,\ \omega'_2 = 1\}.$$

This σ-field consists of 2^{2^2} elements (see below). The σ-field $\mathcal{F}_N$ is generated by the 2^N one-element subsets of Ω, since $A_\omega = \{\omega\}$ in this case. Thus, $\mathcal{F}_N = 2^\Omega$ is the whole original σ-field.

Since the set of generators for $\mathcal{F}_m$ is independent under taking their unions (the unions of two distinct collections of generators are distinct subsets of Ω), the total number of the elements of $\mathcal{F}_m$ is the same as the number of subsets of the set of the generators, i.e. 2^{2^m} (as Exercise 2.12 says). Since the generators for $\mathcal{F}_m$ with $m < N$ are obtained from the generators for $\mathcal{F}_{m+1}$ by replacing the pairs A_ω and $A_{\omega'}$ of the latter generators corresponding to paths ω and ω' of $m+1$ steps differing in the last step by $A_\omega \cup A_{\omega'}$, the generators for $\mathcal{F}_m$ are contained in $\mathcal{F}_{m+1}$. Thus, $\mathcal{F}_m \subset \mathcal{F}_{m+1}$ and we obtain a filtration

$$\{\emptyset, \Omega\} = \mathcal{F}_1 \subset \mathcal{F}_2 \subset \ldots \subset \mathcal{F}_{N-1} \subset \mathcal{F}_N = 2^\Omega,$$

as Exercise 2.13 and the sentence immediately after say.

However, the second-to-last paragraph on p50 would seem to fit more naturally with *fixing* a path $\omega = (\omega_1, \dots, \omega_m)$ of $m \le N$ steps and taking $\mathcal{F}_\omega = 2^{A_\omega}$ to the set of all subsets of A_ω. This is a σ-field on A_ω, but not on Ω. It consists of $2^{2^{N-m}}$ elements and describes the events that can occur after the first m steps are known. The wording in the paragraph above Exercise 2.14 and in this exercise is nominally correct, but appears similarly artificial.

pp57-58: Definition 3.1 and Theorem 3.3 need to be stated for $\overline{\mathbb{R}}$ instead of $\mathbb{R}$, as used in Theorem 3.9 and Definition 3.14, for example. The statement of Theorem 3.3 should then have the closed parenthesis] around ∞ and [around $-\infty$. The proof then requires an extra step at the end to pass to the interval appearing in the present statement.

p64, Theorem 3.9: the first two statements are not used in the proof and are special cases of the middle two statements
p65, Theorem 3.12: the set $\{x \in E : f(x) \neq g(x)\}$ is null

p65/p289, Exercise 3.5: the statement is correct, but the proof needs to deal with the possibility that

$$g(x) \equiv \liminf f_n(x) \qquad \text{and} \qquad h(x) \equiv \limsup f_n(x)$$

take values of ∞ or $-\infty$ simultaneously, in which case $h - g$ is not defined.

p67, first displayed eqn: it would make more sense to write

$$X^{-1}(\mathcal{B}) = \big\{X^{-1}(B) \colon B\in\mathcal{B}\big\}.$$

Since X is measurable, $X^{-1}(\mathcal{B})\subset\mathcal{F}$ automatically.
p68, last displayed eqn: the definition of what this means is the first displayed eqn on p69
p69, Example 3.17: *12 am* $\longrightarrow$ *12 noon*

p70 bottom - p73 top: Section 3.5.5 is messed up even worse than the portion of Section 2.6.3 starting at the bottom of p50. Some sense can at least be made of the first paragraph of this section. Apparently each $S(n)$ is an $\mathbb{R}$-valued function on some set Ω of stocks. A *European call option at strike price K at exercise time N* gives you the option of buying a stock ω at price K at time N. You will do so if its price $\{S(N)\}(\omega)$ exceeds, so the payoff function will be

$$\big\{S(N)-K\big\}^+ \colon \Omega \longrightarrow \mathbb{R}, \qquad \big\{S(N)-K\big\}^+(\omega) = \max\big\{\{S(N)\}(\omega)-K, 0\big\}.$$

The total value of the option is some weighted sum of the values of this function over all $\omega\in\Omega$ if Ω is finite and some integral otherwise.

The proofs of Propositions 3.21 and 3.24 depend on Proposition 4.15. The *cash payments* $X(n)$ are apparently again some $\mathbb{R}$-valued functions on Ω. The first displayed equation on p71 does not make sense without a base case, e.g. $X(0)$, specified. Definition 3.2 depends on the notion of a Borel subset of $\mathbb{R}^n$, which does not appear until Chapter 6. Example 3.26 depends on Section 7.4.4. In the second displayed equation on p73, p_* should be p. The symbols r, U, and D refer to the *risk-free interest rate* as on p71 and factors of change in the stock price as on p50. An *American put option with strike price K and expiration time N* gives you the option of selling a stock ω at any time $n=1,2,\ldots,N$ (provided you had not sold it earlier). If you do so, your payoff will be

$$g\equiv\big\{K-S(n)\big\}^+ \colon \Omega \longrightarrow \mathbb{R}, \qquad g(\omega) = \max\big\{K-\{S(n)\}(\omega), 0\big\}.$$

With this, the formulas in the top half of p73 make sense.

p77, Exercise 4.1(c): a different version of this exercise appears in the official erratum, but the solution in the book is for the present version and is not corrected in the erratum

pp77-80: the proof of Theorem 4.6(i) involves circular logic. The proof of Proposition 4.5 in Section 4.8 first deduces the statement of Theorem 4.6(i) directly from Definition 4.2 (without saying so). So, it makes no sense to claim that Theorem 4.6(i) is a consequence of Proposition 4.5. Theorem 4.6 is only about simple functions and so should appear before Definition 4.4.

Before proceeding past Exercise 4.1, one should actually check that Definition 4.2 on p76 is independent of the presentation of φ as a sum in the preceding equation. In other words, suppose

$$\varphi\equiv\sum_{i=1}^{k} a_i\mathbb{1}_{A_i} \qquad \text{and} \qquad \psi\equiv\sum_{j=1}^{\ell} b_j\mathbb{1}_{B_j} \tag{1}$$

are simple (nonnegative) functions as in Definition 4.1. One then needs to show that

$$\sum_{i=1}^{k} a_i m\big(A_i \cap E\big) = \sum_{j=1}^{\ell} b_j m\big(B_j \cap E\big).$$

The condition $\varphi = \psi$ does not imply that $k = \ell$, $a_i = b_i$, and $A_i = B_i$. It does imply that if $A_i \cap B_j \neq \emptyset$, then $a_i = b_j$. This means that

$$\begin{aligned}\sum_{i=1}^{k} a_i m\big(A_i \cap E\big) &= \sum_{i=1}^{k} a_i m\bigg(\bigcup_{j=1}^{\ell} B_j \cap A_i \cap E\bigg) = \sum_{i=1}^{k}\sum_{j=1}^{\ell} a_i m\big(A_i \cap B_j \cap E\big) \\ &= \sum_{j=1}^{\ell}\sum_{i=1}^{k} b_j m\big(A_i \cap B_j \cap E\big) = \sum_{j=1}^{\ell} b_j m\bigg(\bigcup_{i=1}^{k} A_i \cap B_j \cap E\bigg) = \sum_{j=1}^{\ell} b_j m\big(B_j \cap E\big).\end{aligned}$$

The first and last equalities above hold because the union of B_j is X and the union of A_i is X, respectively. The second and penultimate equalities hold because $B_j \cap B_{j'} = \emptyset$ if $j \neq j'$ and $A_i \cap A_{i'} = \emptyset$ if $i \neq i'$, respectively. The middle inequality above holds because $a_i = b_j$ whenever $m(A_i \cap B_j \cap E) \neq 0$.

It would make sense to then show that if φ and ψ are simple (nonnegative) functions as in (1), then

$$\int_E (\varphi + \psi) = \int_E \varphi + \int_E \psi \tag{2}$$

for the integrals as in Definition 4.1. Since the union of B_j is X and the union of A_i is X,

$$\varphi = \sum_{i=1}^{k}\sum_{j=1}^{\ell} a_i \mathbb{1}_{A_i \cap B_j}, \quad \psi = \sum_{j=1}^{\ell}\sum_{i=1}^{k} b_j \mathbb{1}_{A_i \cap B_j}, \quad \varphi + \psi = \sum_{i=1}^{k}\sum_{j=1}^{\ell} (a_i + b_j) \mathbb{1}_{A_i \cap B_j}.$$

Since each of the collections $\{A_i\}$ and $\{B_j\}$ partitions X, so does the collection $\{A_i \cap B_j\}$. Thus, the sums above are presentations of φ, ψ, and $\varphi + \psi$ as in the equation above Definition 4.1 and so

$$\int_E \varphi = \sum_{i=1}^{k}\sum_{j=1}^{\ell} a_i m\big(A_i \cap B_j \cap E\big), \quad \int_E \psi = \sum_{i=1}^{k}\sum_{j=1}^{\ell} b_j m\big(A_i \cap B_j \cap E\big),$$
$$\int_E (\varphi + \psi) = \sum_{i=1}^{k}\sum_{j=1}^{\ell} (a_i + b_j) m\big(A_i \cap B_j \cap E\big).$$

This establishes (2).

The proof of (2) in the book is postponed to Step 2 in the proof of Theorem 4.19 in Section 4.3. Theorem 4.7(v) for $f = \varphi$ as in Definition 4.1 follows immediately from (2) with φ replaced by $\mathbb{1}_A f$ and ψ by $\mathbb{1}_g f$. This is the conclusion of the first step in the proof of the full statement of Theorem 4.7(v).

p80, proof of (ii): this follows from (i) with $\mathbb{1}_B f \le f$; (ii) is an immediate consequence of (v)
p82, Hint: only Thm 4.7 (i) + (iii) are needed; Prp 4.9 should appear before Thm 4.8
p82, Proposition 4.10: this has no connection to the present section or even the next one, both of which are about nonnegative functions. This is used in Section 4.2, but this result was already established in Section 3.4. As the paragraph above Prp 4.10 suggests, Theorem 3.9 provides another proof that $f^{\pm}$ is measurable if f is, but this is not the proof appearing in the appendix.

p82, Theorem 4.11: it would be more logical to flip the two sides of the inequality because one is usually interested in studying the integral of a limit of a sequence of functions, not so much a limit of their integrals. This would also fit better with the proof and the use of this statement in the proof of Theorem 4.13.

p83, 1st equation: either the first line case should be $x \in E$ and $\varphi(x) > 0$ or the second one should be just $\varphi(x) > 0$
p83, 3-line displayed equation: 3rd line is missing *for* $k \ge n$
p84, Theorem 4.13: same comment as for p82, Theorem 4.11
p86, Proposition 4.15: need to add $\infty 1_{f^{-1}(\infty)}$ to the book's expression for s_n
p86, Definition 4.16: *integrable* $\longrightarrow$ *integrable on E*

p87, displayed equation after Exercise 4.5: this statement is not trivial. It follows immediately from the conclusion of Step 3 in the proof of Theorem 4.19 on page 89 because $|f| = f^+ + f^-$ is a sum of nonnegative functions. It can also be deduced from Theorem 4.7(v) with

$$A = \{x \in E \colon f^+(x) > 0\} = \{x \in E \colon f(x) > 0\}, \quad B = \{x \in E \colon f^-(x) > 0\} = \{x \in E \colon f(x) < 0\}$$

because these two sets are disjoint, $|f| = f_+$ on A, and $|f| = f_-$ on B.

p90, Proposition 4.20: For $c \ge 0$ and $f \ge 0$, this is Theorem 4.7(iii). The general case follows from this case and Definition 4.16.

p90, Theorem 4.21: the two statements in the proof are contained in the statements of Theorem 4.19 and of Proposition 4.20

p90, Theorem 4.22: f, g need to be integrable on $\mathbb{R}$. The proof needs to make use of Proposition 4.23(i) because $h = g - f$ may not be defined everywhere.

p91, Proposition 4.23: (i) is straightforward, is needed for Theorem 4.22, and should appear shortly after Definition 4.16; (ii) is an immediate consequence of Proposition 4.17 and should appear right after it; (iii) is an immediate consequence of the displayed equation on p87 and should appear there; (iv) is Theorem 4.8 in Section 4.1 and does not belong in this section; f needs to be integrable on A, not just measurable in (ii) and (iii).

p91, Theorem 4.24: this belongs at the end of Section 4.2
p92, Example 4.25: *for all* $x \neq 0$

p92, Theorem 4.26: same comment as for p82, Theorem 4.11
p93, below (4.2): $g - f_n$ is not defined at x such that $g(x) = f_n(x) = \infty$. However, the subset of such points of E is of measure 0 by Proposition 4.23(i); it can thus be excluded without affecting any integrals. A similar comment applies to the function $f_n + g$ at the bottom of the page.

p94, Example 4.27: The comment in the parenthesis at the end of the example is wrong because the function $1/\sqrt{x}$ is not bounded on $(0, 1)$. Therefore, Theorem 4.33 does not apply. However, Theorem 4.36 does apply.

p94, Proposition 4.28: The functions h_n are *not* bounded unless f is bounded below. On the other hand, one could truncate similarly at $-n$.

p95, Theorem 4.30: need f_k measurable; in the remainder of this section the summation index is n, not k

p98, Theorem 4.33: the first sentence should appear inside (i), since it does not apply to (ii). In (ii), *bounded* is missing before *functions*

p98, below (4.3): $M = \sup\{|f(x)| : x \in [a, b]\}$
p101, Theorem 4.36: *always* $\longrightarrow$ *also*

pp102-104, Theorems 4.38 and 4.39: The argument would be clearer if the two steps in the proof of Theorem 4.38 were separated into two statements and the splitting of f into f_+ and f_- were carried out in the first step. The first step, i.e. approximating an integrable function by a step function, is applicable to any measure space $(X, \mathcal{F}, \mu)$. It would be simpler to carry out the second step, i.e. approximating a step function by a step function based on intervals, first for $\varphi = a\mathbb{1}_A$; the general case is obtained by taking the sum of the functions for each $a_i\mathbb{1}_{A_i}$ (or $a_n\mathbb{1}_{A_n}$ in the notation of Definition 4.1). The same applies to the proof of Theorem 4.39.

The second step in the proof of Theorem 4.38 is applicable to any measure μ which is a restriction of some outer measure μ^* constructed on 2^X as in Definition 2.3 with *the intervals* replaced by some special subsets of 2^X on which measure is defined (e.g. rectangles in $\mathbb{R}^2$). It can be established directly from Definition 2.3, instead of going via Theorem 2.17. The proof of Theorem 4.38 does not provide disjoint intervals $I_n = G_i$, though this is not necessary if the proof of Theorem 4.39 were modified as suggested above. There is no reason to restrict to $[a, b]$ in Theorem 4.38; this would eliminate most of the proof of Theorem 4.39.

pp104,105, Lemma 4.40 and its proof: all integrals should be Lebesgue, not Riemann. The statement concerning g should be separated out from the proof and done before Lemma 4.40. It would make more sense to present the first displayed equation on p105 in the reverse order.

p105,106 Theorems 4.41,4.42: Neither of these statements requires restricting to probability spaces. Theorem 4.41 is a change-of-variables for Lebesgue integrals and applies to an arbitrary measure space $(X, \mathcal{F}, \mu)$ in place of the probability space $(\Omega, \mathcal{F}, P)$. It requires some conditions on g and X to make sure that $g\circ X$ is measurable function. This is the case if either

- g is Borel (not just Lebesgue) measurable and X is measurable or
- g is measurable and $X^{-1}(E)\in\mathcal{F}$ for all $E\in\mathcal{M}$ (if X is measurable, then $X^{-1}(E)\in\mathcal{F}$ for all $E\in\mathcal{B}\subset\mathcal{M}$).

This condition should have appeared in the statement in the book as well. Theorem 4.42 applies to any countable collection of measures μ_i (instead of P_i) on any σ-field $\mathcal{F}$ on any set X and to any collection $p_i\in\mathbb{R}^{\ge 0}$. The additional conditions in the book ensure that $(\Omega,\mathcal{F},P_X)$ is a probability space, but this is irrelevant for the stated formula. It also makes little sense to use the subscript X, since there is no X in the statement.

p109, Example 4.43: $\big[\frac{1}{3},\frac{2}{3}\big),\ \big[\frac{1}{9},\frac{2}{9}\big),\ \big[\frac{7}{9},\frac{8}{9}\big] \ \longrightarrow\ \big[\frac{1}{3},\frac{2}{3}\big],\ \big[\frac{1}{9},\frac{2}{9}\big),\ \big[\frac{7}{9},\frac{8}{9}\big]$
p110, Proposition 4.44(ii): the reverse order of the statements would be more logical
p110, Exercise 4.15(c) and below: measures on subsets of $\mathbb{R}^n$ with $n>1$ are not defined until Chapter 6. Theorem 4.45 does not deal with subsets of spaces other than $\mathbb{R}$.

pp111,112, Theorem 4.45 and its proof: The statement is wrong and the proof is messed up. The function X can take values $-\infty$ (at $\omega=0$) and ∞ (at $\omega=1$). This happens if F is a *strictly* increasing function which approaches 0 as $x\longrightarrow-\infty$ and approaches 1 as $x\longrightarrow\infty$, such as

$$F\colon\mathbb{R}\longrightarrow[0,1], \qquad F(x)=\frac{1}{1+\mathrm{e}^{-x}}\,.$$

Thus, $X\colon[0,1]\longrightarrow\mathbb{R}$ in the statement should be replaced by $X\colon[0,1]\longrightarrow\overline{\mathbb{R}}$.

It would be more logical to define X^- first and X^+ second,

$$\begin{aligned} X^-\colon[0,1]&\longrightarrow\overline{\mathbb{R}}, & X^-(\omega)&=\sup\big\{x\in\mathbb{R}\colon F(x)<\omega\big\},\\ X^+\colon[0,1]&\longrightarrow\overline{\mathbb{R}}, & X^+(\omega)&=\inf\big\{x\in\mathbb{R}\colon F(x)>\omega\big\},\end{aligned}$$

at the beginning of the proof, since X^- is about something on the left and X^+ is about something on the right. Furthermore, most of the proof involves X^-; X^+ comes in only at the very end, as a secondary observation *after* the statement of Theorem 4.45 is established. The captions of Figures 4.8-4.10 do not fit with the figures. These figures show the three possible types of behavior of F around some point $y\in\mathbb{R}$, which affects what $X^\pm(\omega)$ does around $\omega=F(y)$.

It is immediate from the definitions of $X^\pm$ that

$$\begin{gathered} X^-(0)=-\infty,\quad X^+(1)=\infty,\quad X^\pm(\omega)\le X^\pm(\omega')\ \ \forall\,\omega,\omega'\in[0,1],\,\omega\le\omega'\,,\\ X^+(\omega)\le y\quad\forall\,\omega\in[0,F(y)),\ y\in\mathbb{R}. \end{gathered}\tag{3}$$

This in turn implies that for every $y\in\mathbb{R}$ the sets

$$\big\{\omega\in[0,1]\colon X^-(\omega)\le y\big\} \qquad\text{and}\qquad \big\{\omega\in[0,1]\colon X^+(\omega)\le y\big\}$$

are intervals. The first of these intervals contains 0; the second interval either contains 0 or is empty (depending on y). Thus,

$$F_{X^-}\colon \mathbb{R}\longrightarrow[0,1],\quad F_{X^-}(y)\equiv m\big(\{\omega\in[0,1]\colon X^-(\omega)\le y\}\big)=\sup\{\omega\in[0,1]\colon X^-(\omega)\le y\}.$$

Furthermore, the function F_{X^-} and the function

$$F_{X^+}\colon \mathbb{R}\longrightarrow[0,1],\quad F_{X^+}(y)\equiv m\big(\{\omega\in[0,1]\colon X^+(\omega)\le y\}\big),$$

are non-decreasing. By (3),

$$F(y)\le F_{X^+}(y)\,. \tag{4}$$

The assumption of Proposition 4.44(i), i.e. that F is non-decreasing, implies that

$$X^-(\omega)\le X^+(\omega)\ \ \forall\,\omega\in[0,1],\qquad X^-\big(F(y)\big)\le y\le X^+\big(F(y)\big)\ \ \forall\,y\in\mathbb{R}.$$

Along with the definitions of F_{X^-} and F_{X^+}, these inequalities give

$$F(y),F_{X^+}(y)\le F_{X^-}(y)\quad\forall\,y\in\mathbb{R}\,. \tag{5}$$

The assumption of Proposition 4.44(i) together with Proposition 4.44(iii), i.e. that F is continuous from the right, imply that

$$y<X^-\big(F(x)\big)\quad\forall\,y,x\in\mathbb{R}\text{ s.t. }F(y)<F(x). \tag{6}$$

The first assumption of Proposition 4.44(ii), i.e. that $F(x)\longrightarrow1$ as $x\longrightarrow\infty$, implies that

$$\big\{x\in\mathbb{R}\colon F(y)<F(x)<\omega\big\}\neq\emptyset\quad\forall\ \omega\in(F(y),1],\ y\in\mathbb{R}\text{ s.t. }F(y)<1. \tag{7}$$

By the definition of X_-, (6), and (7),

$$F_{X^-}(y)\le F(y)\quad\forall\,y\in\mathbb{R}.$$

Combining this with the first inequality in (5), we find that $F=F_{X^-}$. This establishes Theorem 4.45, with the correction described above, i.e. that $F=F_X$ for some function $X\colon[0,1]\longrightarrow\overline{\mathbb{R}}$.

Combining the conclusion of the above paragraph, i.e. that $F=F_{X^-}$, with (4) and the second inequality in (5), we obtain $F=F_{X^+}$. This is the claim of the last paragraph of the "proof of Theorem 4.45". However, the argument in the book is completely wrong. The second equality in the last, very long equation (which should have clearly been displayed, instead of being in text) is equivalent to the statement that

$$F_{X^+}(q)\equiv m\big(\{\omega\in[0,1]\colon X^+(\omega)\le q\}\big)=F(q).$$

However, this is precisely the statement to be proved!!!

The argument above does not use the second condition of Proposition 4.44(ii), i.e. that $F(x)\longrightarrow0$ as $x\longrightarrow-\infty$. It is satisfied if $X:\Omega\longrightarrow\mathbb{R}$, but such X are insufficient for the statement of Theorem 4.45 as noted above. It is also satisfied if $P(X^{-1}(\infty))=0$. If F in Theorem 4.45 satisfies this condition, then the function X^- defined above takes values in $\mathbb{R}$ on $(0,1]$, but $X^-(0)=-\infty$ (which in general is unavoidable as noted above).

p112, last paragraph: Directly contrary to what this paragraph says, the author's formulations only for probability obfuscate the situation and do not simplify the notation. The next two comments illustrate this.

p113, Theorem 4.46: The statement means that $f_X:\mathbb{R}^n\longrightarrow[0,\infty]$ is an integrable function and

$$P_X(A)=\int_A f_X \mathrm{d}m_{\mathbb{R}^n} \tag{8}$$

for every Borel measurable subset $A\subset\mathbb{R}^n$, where $m_{\mathbb{R}^n}$ is the Lebesgue measure. This statement has nothing to do with any random variable X (which should thus be omitted from the notation) and is applicable to every measure space $(X,\mathcal{F},\mu)$ in place of $(\mathbb{R}^n,\mathcal{M}_{\mathbb{R}^n},m_{\mathbb{R}^n})$. The proof applies verbatim to this general case as well. On the other hand, the Lebesgue measure on $\mathbb{R}^n$ is not defined until Chapter 6.

p113, Corollary 4.47: As stated, this makes no sense. The intended meaning is that $X:\Omega\longrightarrow\mathbb{R}^n$ is as in Theorem 4.41, but with $\mathbb{R}$ replaced by $\mathbb{R}^n$ (and with the additional condition described in the comment on Theorem 4.41 above), and f_X satisfies (8). The claim then indeed follows from Theorems 4.41 and 4.46, but with $\mathbb{R}$ replaced by $\mathbb{R}^n$ in the former.

p113, Theorem 4.48: The statement is imprecise, and the proof does not directly address it. The missing assumptions are that g is *continuously* differentiable, $g'(x)\neq 0$ for all $x\in\mathbb{R}$ (this is not implied by g being increasing), $X:\Omega\longrightarrow\mathbb{R}$ is a measurable function, and $f_X:\mathbb{R}\longrightarrow[0,\infty]$ is an integrable function such that

$$P_X(A)\equiv P\big(X^{-1}(A)\big)=\int_A f_X \mathrm{d}m \tag{9}$$

for every Borel measurable subset $A\subset\mathbb{R}$. The claim is that

$$P_{g\circ X}(A)\equiv P\big(X^{-1}\big(g^{-1}(A)\big)\big)=\int_A (f_X\circ g^{-1})(g^{-1})'\mathrm{d}m \tag{10}$$

for every Borel measurable subset $A\subset\mathbb{R}$.

It would have been useful to have an exercise or a proposition early in Section 4.7.2 stating that

$$m_g(A)\equiv m\big(g^{-1}(A)\big)=\int_A (g^{-1})'\mathrm{d}m \tag{11}$$

for every increasing, continuously differentiable function $g:\mathbb{R}\longrightarrow\mathbb{R}$ such that $g'(x)\neq 0$ for all $x\in\mathbb{R}$ and for every measurable subset $A\subset\mathbb{R}$, i.e.

$$f_g=(g^{-1})':\mathbb{R}\longrightarrow(0,\infty)$$

with the definitions as in Corollary 4.47. If A is a bounded integral, (11) is just the Fundamental Theorem of Calculus (Proposition 4.32). Otherwise, A can be approximated by such intervals. Since $(g^{-1})'$ is bounded on bounded intervals, such an approximation provides an approximation for $g^{-1}(A)$. Thus, the general case of (11) follows from the basic case via the Monotone Convergence Theorem.

We can then apply Corollary 4.47 with $\Omega = g^{-1}(A)$, $P = m$, g replaced by $f_X \circ g^{-1}$, and $X = g$. By (9), this corollary, and (11),

$$P\big(X^{-1}(g^{-1}(A))\big) = \int_{g^{-1}(A)} f_X \mathrm{d}m = \int_{g^{-1}(A)} \{f_X \circ g^{-1}\} \circ g \mathrm{d}m$$
$$= \int_A f_g \{f_X \circ g^{-1}\} \mathrm{d}m = \int_A (f_X \circ g^{-1}) g^{-1} \mathrm{d}m \,.$$

This establishes (10).

p114, Example 4.50: The statement means that

$$P_X(A) = \int_A n(x) \mathrm{d}m$$

for every Borel measurable subset $A \subset \mathbb{R}$. The conclusion is that $P_Y = P_{g(X)}$ is given by the above formula, but with $n(x)$ given by (4.6) in the book. In the last sentence, $g^{-1}(y)$ should be $(x-\mu)/\sigma$.

p114, last two equation: these should be Lebesgue, not Riemann, integrals
p115, first equation: $f \longrightarrow f_2$
p116, 3rd line: need to take absolute value of the real part after *but*
p116, middle: the first formula for φ_X (after its definition) requires a generalization of Theorem 4.41 to complex-valued measurable functions X with $\mathbb{R}$ in place of Ω. The second formula follows from the first via Theorem 4.46.

p116, bottom: Proposition 4.23(iii) as stated does not safisfy; one needs its extension to complex-valued functions. This extension is provided by the last equation on p115.

p117, (4.8): $S(0)$ does not appear above
p118, above Proposition 4.56: there is no definition of η
p118, Proposition 4.56: $pU\mathrm{e}^{-rT} \longrightarrow pU\mathrm{e}^{-rT}$
p119, Proposition 4.57: the function $N(d)$ is defined in the proof and incorrectly

p121, proof of Proposition 4.28: $g_n \leq |f| \longrightarrow |g_n| \leq |f|$; $h_n \leq |f| \longrightarrow |h_n| \leq |f|$
p122, first displayed equation: Monotone Convergence Theorem is used in the first equality
p123, proof of Proposition 4.56: $p^k(1-q)^{N-k} \longrightarrow p^k(1-p)^{N-k}$; $PU\mathrm{e}^{-rT} \longrightarrow pU\mathrm{e}^{-rh}$
$1-q = (1-p)U\mathrm{e}^{-rT} \longrightarrow 1-q = (1-p)D\mathrm{e}^{-rh}$; Ψ is already defined in the statement
p124: the last two displayed equations are missing factors of $\sqrt{2\pi}$; N is not the cumulative distribution as defined on p109; in the second-to-last equation, the lower limit should be d–$\sigma\sqrt{T}$

pp126-145: It would have made more sense to combine Sections 5.1, 5.3, and 8.1 and put Section 5.2 after that. This would have avoided a number of repetitions throughout this chapter (such as separate proofs of completeness for L^1 and L^p and separate proofs of Minkowski's inequality for L^2 and L^p).

p126, top: this refers to the beginning of Section 4.7.4
p128, line 4: $L_1(E) \longrightarrow L^1(E)$; $\mathcal{L}_1(E) \longrightarrow \mathcal{L}^1(E)$
p129, Example 5.2(bc): these two sequences are not even in $L^1(0,\infty)$
p130, Theorem 5.5, proof: in the preceding argument, N_i was n_i
p131, (5.1): $<\varepsilon \longrightarrow \le\varepsilon$
p134, Exercise 5.3: $L^2(R) \longrightarrow L^2(\mathbb{R})$
p134, Exercise 5.4(bc): these two sequences are not even in $L^2(0,\infty)$
p137, Exercise 5.6: For f and g as given, $\|f\|_1^2=\|g\|_1^2=1/16$, not $1/4$ as stated in the solution on p293. These two functions thus do not provide a contradiction to the parallelogram law. One could use $f=1$ and $g=2x$ instead, which would give

$$\|f+g\|_1^2 = 4, \quad \|f-g\|_1^2 = \frac{1}{4}, \quad \|f\|_1^2 = 1, \quad \|g\|_1^2 = 1\,.$$

p138, middle: $\|h_n=h\| \longrightarrow \|h_n-h\|$

p139, top: This part is intended to establish the equivalence statement, i.e. that $h''\equiv h-h'$ is orthogonal to every element of K if and only if

$$\|h-h'\| = \inf\big\{\|h-k\|: k\in K\big\}.$$

This important statement does not depend on K being complete and should have been its own lemma. The proof of the converse in the book is wrong if H is a Hilbert space over $\mathbb{C}$, as is assumed. It shows only that $(h'',k)\in\mathfrak{i}\mathbb{R}$ for all $k\in K$. However, $(h'',\mathfrak{i}k)=-\mathfrak{i}(h'',k)$. Thus, $(h'',k),(h'',\mathfrak{i}k)\in\mathfrak{i}\mathbb{R}$ implies that $(h'',k)=0$.

Most of the last paragraph of this proof shows that if $k_n\in K$ is a sequence such $\|h-k_n\|$ converges to the above infinimum, then the sequence k_n is Cauchy. This important statement does not depend on K being complete either and should have been its own lemma also. The argument would have been notationally simpler of the Parallelogram Law (Proposition 5.12(i)) were applied to the vector $h-k_m$ and $h-k_n$.

p140, bottom: Definition 3.14 is not restricted to nonnegative functions. What the book defines here as the essential supremum of f is the essential supremum of $|f|$. On the first line of Definition 5.18, it is indeed written as the essential supremum of $|f|$, but on the fourth line it becomes the essential supremum of f again.

p141, Lemma 5.20: Writing this statement in terms of $p,q\in(1,\infty)$ with $1/p+1/q=1$ instead of α,β would have been more convenient for its application in Theorem 5.21. Using something else for x,y, such as u,v or a,b, would have been more consistent with the usage of the lemma as well, since x,y are usually used as independent variables and are replaced by $|f|^p$ and $|g|^q$

in the proof of Lemma 5.21.

p142, Theorem 5.21, 2nd displayed equation in the statement: $\overline{g} \longrightarrow g$

p143, Theorem 5.23, proof: The inequality in the second displayed equation holds only almost everywhere. It is also not relevant. On the other hand, it would have been useful to note that the expression in the third displayed equation is $\|f+g\|_p^{p/q}$ and that it is finite. Using Sobolev norms after the first line in the last displayed expression would have made the argument clearer and taken less space.

p145, Exercise 5.8: this sequence is not even in $L^4(0,\infty)$
p145, Theorem 5.25: use of Hölder's inequality would have been more instructive

p146, Definition 5.26: This should read *For* $n=1,2,\ldots$, *the moment of order* n *of a random variable* $X\in L^n(X)$ *is the number* $\mathbb{E}(X^n)$, instead of having $n=1,2,\ldots$ at the end. The restriction that $X\in L^n(X)$ is inconsistent with the usage in Proposition 5.27 and Exercise 5.9, which allow infinite moments. The second displayed equation should have $\mathbb{E}((X-\mu)^n)$ instead of $\mathbb{E}(X-\mu)^n$. On the next line, it should be *the moments*.

p147, first equation: $\mathbb{E}(X-\mathbb{E}(X))^2 \longrightarrow \mathbb{E}((X-\mathbb{E}(X))^2)$
p147, Example 5.31: this was Exercise 4.18(b)
p149, above Example 5.34: $\mathbb{E}(X-2\mu)^{2k} \longrightarrow \mathbb{E}((X-2\mu)^{2k})$
p150, Theorem 5.36, proof: Using B and C instead of B_1 and B_2 would be more consistent with Definition 3.18. So would be writing $P(X^{-1}(B_1)\cap Y^{-1}(B_2))$ on the right-hand side of the second displayed equation in the proof.

p151, Definition 5.38: *centred* $\longrightarrow$ *centered*
p151, above Example 5.39: need to take some approximation for f as in the proof of Prp 5.37
p151, last line: $\mathbb{E}(XY) = \longrightarrow \mathbb{E}(XY) = \frac{1}{2}$
p152, line 5: $P(X^{-1}(A)\cap Y^{-1}(A)) = \frac{1}{2}$
p152, above Proposition 5.41: $(\mathbb{E}X)^2 \longrightarrow (\mathbb{E}(X))^2$
p156, first equation: $\mathbb{E}(X-\mu)^n \longrightarrow \mathbb{E}((X-\mu)^2)$
p156, proof of Proposition 5.40: 9 missing pairs of parenthesis involving $\mathbb{E}$
p156, first line: *repetitive use of the formula for two* is *induction* done informally
p156, last line: *step-by-step application of the result for two* is *induction* done informally

pp159-173, Sections 6.1-6.4: This part is very poorly organized, establishes the main result only for finite measures, instead of σ-finite, and does not even state Fubini's theorem for the standard Lebesgue measure on $\mathbb{R}^2$; pp160-170 in Rudin's *Real and Complex Analysis* do a much better job. Theorem 6.2 is not needed for anything. The sentence in the proof of Theorem 6.5 after the first displayed equation implicitly uses that the collection $\mathcal{A}$ of finite unions of disjoint rectangles is a field; this is not stated explicitly until the bottom of p168 (well after the proof of Theorem 6.5) and is not completely obvious. The proof of Theorem 6.5 for an arbitrary element of $\mathcal{A}$ of finite disjoint unions of measurable rectangles is omitted as *easy but tedious*. It would have been automatically established by showing that the collection $\mathscr{C}$ is preserved under finite unions; this is easy and does not even require the page taken up by the proof already. The statement of Fubini Theorem, Theorem 6.13, itself is *wrong*: the section functions of Theorem 6.10 are always measurable, but are integrable only for almost all values of the parameter (because the corresponding +/− parts could be infinite at some values of the parameters).

p161, Theorem 6.2(ii): The notion of measurable function has been defined only for functions with values in $\mathbb{R}$ and $\mathbb{C}$. What is meant by measurable function here is that $\mathrm{Pr}_i^{-1}(A)$ is a measurable subset of the domain for every measurable subset A of the target. This fits with the notion of measurable functions with values in $\mathbb{R}$ if one takes the σ-field of Borel (instead of Lebesgue) measurable subsets on the target $\mathbb{R}$ (preimages of Lebesgue sets under functions measurable in this sense need not be measurable).

p164, lines 4,6: $P(A_{\omega_2}) \longrightarrow P_1(A_{\omega_2})$; $P(A_1) \longrightarrow P_1(A_1)$
p164, above Definition 6.6: $\mathcal{F} \longrightarrow \mathcal{F}_{\mathcal{A}}$

p164, above Definition 6.6: The actual definition of a field $\mathcal{A}$ on X is that $X \in \mathcal{A}$ and $\mathcal{A}$ is closed under complements (if $A \in \mathcal{A}$, then $A^c \in \mathcal{A}$) and *finite* unions (if $A, B \in \mathcal{A}$, then $A \cup B \in \mathcal{A}$). This implies that $\mathcal{A}$ is closed under differences (if $A, B \in \mathcal{A}$, then $A - B \in \mathcal{A}$). The definition in the book, made in passing, is missing the condition that $X \in \mathcal{A}$.

p164, Definition 6.4: *family* $\longrightarrow$ *collection*
p166, 2nd paragraph: this contains a general statement that a monotone class which is a field is also σ-field. This should be a separate lemma.
p166, above proof of Theorem 6.5: *family of all unions* $\longrightarrow$ *collection of finite unions*

p168, proof of Theorem 6.8: The penultimate equality in the first displayed equation uses the Monotone Convergence Theorem, not Beppo-Levi. An application of Beppo-Levi requires verifying that a certain sum is finite. This is true in this case, but this approach does not extend to the σ-finite case (the Monotone Convergence Theorem does extend). The end of this proof on p169 uses the finiteness of the measures.

p169, paragraph above Section 6.4: This refers to the product measure on $\mathbb{R}^2$, but $(\mathbb{R}, \mathcal{M}, m)$ is σ-finite and not finite. The key Theorems 6.5 and 6.8 in this section are stated and proved only for products of finite measures.

p170, proof of Theorem 6.10: the claim is immediate from the definition of measurable function via Theorem 6.4. The proof does not require any approximations or the nonnegativity assumption.

p171, Remark 6.14: this is out of place and belongs with the last paragraph of Section 6.3.

p173, bottom: *joint distribution* $\longrightarrow$ *joint distribution* $P_{X,Y}$
p174, Example 6.17: the claim applies to every smooth map $(X, Y)\colon [0,1] \longrightarrow \mathbb{R}^2$
p174, last paragraph: this has no relevance here and should appear just before Corollary 6.22

p175, above Exercise 6.6: no need for *differentiation*; claim follows from definition on p109
p176, proof of Theorem 6.20: the long math expression should be displayed
p177, Corollary 6.22: *orthogonal* is $\rho=0$ at the bottom of p174
p189, Proposition 7.2: only ν needs to be finite
p189, bottom: $f:\Omega\longrightarrow\mathbb{R} \;\longrightarrow f:\Omega\longrightarrow\mathbb{R}^{\geq 0}$
p191, (b): swapping the two sides of the equality would make the relation with (a) clearer
p193, Theorem 7.7, proof: $h_v \;\longrightarrow h_\nu$
p194, Proposition 7.9(ii): need $\lambda\ll\nu$
p197, bottom: $h(\omega<1 \;\longrightarrow h(\omega)<1$

p198, top, hint: Proposition 7.11 cannot be used directly without allowing negatively valued measures. However, its proof can be used.

p235, proof of Proposition 7.2: $\nu(F_1) \;\longrightarrow \nu(E_1)$; (i) $\longrightarrow$ (ii)
p237, proof of Proposition 7.9(ii): this follows immediately from Proposition 7.6
pp241-243, Section 8.1: this should appear immediately after Sections 5.1 and 5.3

pp241,242, Definition 8.1: In Chapter 5, there are norms $\|\cdot\|_\infty$ and $\|\cdot\|_p$ for $[p,\infty)$. The latter is precisely what is written as $|f_n-f|_p$ in (iv). The former is the essential supremum and thus is weaker than what $|\cdot|_\infty$ appearing in (i), but their $|\cdot|_\infty$ is a norm on $\mathcal{L}^\infty$ which does not descend a norm on L^∞ because it distinguishes between functions differing only on measure 0 sets.

p289, 3.5: cannot take the difference when $f_n(x)=g_n(x)=\infty$
p291, 4.8: *for* $x\geq 0 \;\longrightarrow$ *for* $x>0$
p293, 5.6: $\|f\|_1^2=\frac{1}{16}$, $\|g\|_1^2=\frac{1}{16}$; there is no contradiction with the parallelogram law
p294, displayed equation: first equality uses the equation below Proposition 5.28
p295, 6.5: first long expression should be displayed, just like the second is
p295, 6.6: $R \;\longrightarrow \mathbb{R}$; it would be more consistent to list the first case as $z\leq 0$ *or* $z\geq 2$ and to skip the last case
p295, 6.6: it would be more consistent to list the first case in the first expression as $z\leq -1$ *or* $z\geq 2$ and as similarly in the second
p295, 6.8: $\frac{1}{3} \;\longrightarrow \frac{1}{4}$; it would have been more systematic and simpler to switch X and Y in the integrand in the second computation (this would make it similar to the first)
p296, 7.2: *union of in* $\longrightarrow$ *union of sets of*