Axiom of Countable Choice

For each $n\in\mathbb{Z}^+$, let A_n be a nonempty set. ACC says that there exists a sequence $(a_n)_{n\in\mathbb{Z}^+}$ with $a_n\in A_n$ for each $n\in\mathbb{Z}^+$. If $\mathbb{Z}^+$ were replaced by a finite set, this would be easily obtainable by induction. According to wiki, a common misconception is that this can be used to establish ACC itself.

One could try to do so as follows. Let

$$\mathcal{C}=\big\{\big(S,(a_n)_{n\in S}\big)\colon S\subset\mathbb{Z}^+,\, a_n\in A_n\ \forall\, n\in S\big\}.$$

Define a partial order $\prec$ on $\mathcal{C}$ by

$$\big(S,(a_n)_{n\in S}\big)\prec\big(S',(a'_n)_{n\in S'}\big)\qquad\text{if}\quad S\subset S'\ \text{ and }\ a_n=a'_n\ \forall\, n\in S. \tag{1}$$

If $(S_k,(a_{k;n})_{n\in S_k})$ is any increasing chain (finite or infinite) of elements of $\mathcal{C}$, define

$$\big(S,(a_n)_{n\in S}\big)\in\mathcal{C}\qquad\text{by}\quad S=\bigcup_k S_k,\quad a_n=a_{k;n}\ \text{ if } n\in S_k. \tag{2}$$

If also $n\in S_\ell$, then $(S_k,(a_{k;n})_{n\in S_k})\prec(S_\ell,(a_{\ell;n})_{n\in S_\ell})$ or the other way around and $a_{k;n}=a_{\ell;n}$ by the definition of $\prec$ in (1). Thus, a_n in (2) is well-defined. Furthermore,

$$\big(S_k,(a_{k;n})_{n\in S_k}\big)\preceq\big(S,(a_n)_{n\in S}\big)\qquad\forall\, k.$$

If S is missing some element $n^*\in\mathbb{Z}^+$, we can choose $a_{n^*}\in A_{n^*}$ and lengthen the above chain by adding

$$\big(S\cup\{n^*\},(a_n)_{n\in S\cup\{n^*\}}\big)\in\mathcal{C}$$

to it. Thus, if we start with a maximal increasing chain of elements of $\mathcal{C}$ (i.e. there is no way of lengthening it), then $S=\mathbb{Z}^+$ and this establishes ACC.

The above is a standard approach to showing that a whole set satisfies some property. It depends on the existence of a maximal increasing chain. Unfortunately, this existence depends on the axiom of choice itself.