MAT 127: Calculus C, Spring 2022
Homework Assignment 7

WebAssign Problems due before 9am, Wednesday, 03/30
20% bonus for submissions **before 9am, Saturday, 03/26**

Written Assignment due before 4pm, Wednesday, 03/30
in your instructor's office (L01 in Math 4-101B, L02/3 in Math 3-111)

Please read Sections 5.2 and 5.3 thoroughly before starting on the problem set; looking over Sections 8.2 and 8.3 of the WebAssign textbook may be helpful too.

Written Assignment: Problems VII.1-6 (below and next page)
Show your work; correct answers without explanation will receive no credit, unless noted otherwise.

Please write your solutions legibly; the graders will disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name (first name first), lecture number (L01, L02, or L03), and HW number in the upper-right corner of the first page.

Problem VII.1

Determine if the series $\sum_{n=1}^{\infty} \ln(n/(n+1))$ converges and find its sum by expressing s_n as a telescoping sum if converges.

Problem VII.2

The Fibonacci numbers are defined in Section 5.1 by $f_0 = 0,\ f_1 = 1,\ f_n = f_{n-1} + f_{n-2}$ if $n \geq 2$. Show that

$$\text{(a)}\ \frac{1}{f_{n-1}f_{n+1}} = \frac{1}{f_{n-1}f_n} - \frac{1}{f_n f_{n+1}}\,, \qquad \text{(b)}\ \sum_{n=2}^{\infty} \frac{1}{f_{n-1}f_{n+1}} = 1\,, \qquad \text{(c)}\ \sum_{n=2}^{\infty} \frac{f_n}{f_{n-1}f_{n+1}} = 2.$$

Problem VII.3

Find the sum of the series $\sum_{n=2}^{\infty} \ln\left(1 - \frac{1}{n^2}\right)$

Problem VII.4

Let f be a continuous positive decreasing function for $x \geq 1$ and $a_n = f(n)$. By drawing a picture, rank $\displaystyle\int_1^6 f(x)\mathrm{d}x$, $\displaystyle\sum_{i=1}^{i=5} a_i$, $\displaystyle\sum_{i=2}^{i=6} a_i$ in increasing order.

Problem VII.5

For what values of p does the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converge?

Problem VII.6

Suppose you have a large supply of bricks, all of the same size, and you stack them at the edge of a table, with each brick extending farther beyond the edge of the table than the one beneath it. Show that it is possible to do this so that the top brick extends entirely beyond the table. In fact, show that the top brick can extend any distance at all beyond the edge of the table if sufficiently many bricks are used.