MAT 127: Calculus C, Spring 2022
Homework Assignment 6

WebAssign Problems due before 9am, Wednesday, 03/23
20% bonus for submissions **before 9am, Saturday, 03/19**

Written Assignment due before 4pm, Wednesday, 03/23
in your instructor's office (L01 in Math 44-101B, L02/3 in Math 3-111)

Please read Sections 5.1 and 5.2 and the Notes on Mathematical Induction thoroughly before starting on the problem set; looking over Sections 8.1 and 8.2 of the WebAssign textbook may be helpful too.

Written Assignment: Problems VI.1-4,G,H (below and next page)
Show your work; correct answers without explanation will receive no credit, unless noted otherwise.

Please write your solutions legibly; the graders will disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name (first name first), lecture number (L01, L02, or L03), and HW number in the upper-right corner of the first page.

Problems VI.1,2

Determine whether each of the following sequences converges. if so, find its limit.

$$(1)\ a_n = \cos(2/n)\,, \qquad (2)\ a_n = \ln(2n^2+1) - \ln(n^2+1)\,.$$

Problem VI.3

Define $\{a_n\}$ by $a_1 = 1$, $a_{n+1} = 1 + 1/(1+a_n)$.
(a) Find the first eight terms of this sequence. What do you notice about the odd terms and the even terms?
(b) By considering the odd and even terms separately, show that the sequence converges and its limit is $\sqrt{2}$.

The above gives the continued fraction expansion

$$\sqrt{2} = 1 + \cfrac{1}{2 + \frac{1}{2+\ldots}}$$

Problem VI.4

Find the sum of the series

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{9} + \frac{1}{12} + \ldots.$$

where the terms are the reciprocals of the positive integers whose only prime factors are 2s and 3s.

Problem G

Show (by induction) that the n-th Fibonacci number f_n defined at the top of p445 is given by

$$f_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right). \tag{1}$$

Is this consistent with the textbook's answer to (3) on p445 and why?

Hint: see *Notes on Mathematical Induction*

Problem H

In the figure below, there are infinitely many inscribed circles in an equilateral triangle, each touching other circles and sides of the triangle. Find the total area enclosed by the circles if the sides of the triangle are of length 1.