MAT 127: Calculus C, Spring 2022
Homework Assignment 5

WebAssign Problems due before 9am, Wednesday, 03/09
20% bonus for submissions **before 9am, Saturday, 03/05**

Written Assignment due before 4pm, Wednesday, 03/09
in your instructor's office (L01 in Math 44-101B, L02/3 in Math 3-111)

Please read Stewart's Section 7.6 thoroughly before starting on the problem set.

Written Assignment: Problems V.1,V.2,F (below and next page)
Show your work; correct answers without explanation will receive no credit, unless noted otherwise.

Please write your solutions legibly; the graders will disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name (first name first), lecture number (L01, L02, or L03), and HW number in the upper-right corner of the first page.

Problem V.1

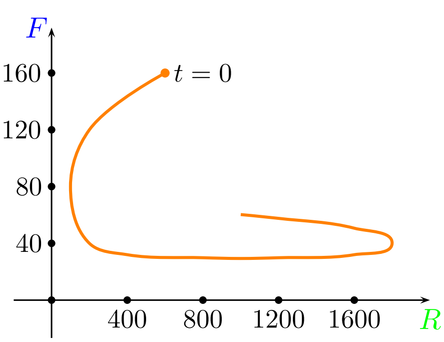
Decide whether each of the following systems of differential equation models two species that compete for the same resources or cooperate for mutual benefit. Explain why each is a reasonable model.

(a) $\begin{cases} \frac{\mathrm{d}x}{\mathrm{d}t} = .12x - .0006x^2 + .00001xy \\ \frac{\mathrm{d}y}{\mathrm{d}t} = .08x + .00004xy \end{cases}$

(b) $\begin{cases} \frac{\mathrm{d}x}{\mathrm{d}t} = .15x - .0002x^2 - .0006xy \\ \frac{\mathrm{d}y}{\mathrm{d}t} = .02y - .00008y^2 - .0002xy \end{cases}$

Problem V.2

A phase trajectory is shown for populations of rabbits (R) and foxes (F) in Figure .
(a) Describe how each population changes as the times goes by.
(b) Use your description to make a rough sketch of the graphs of R and F as functions of time.



Problem F

According to the book, the solutions $(x, y) = (x(t), y(t))$ to the system of differential equations

$$\begin{cases} \frac{\mathrm{d}x}{\mathrm{d}t} = ax - bxy \\ \frac{\mathrm{d}y}{\mathrm{d}t} = -cy + dxy \end{cases} \tag{1}$$

with certain constants $a, b, c, d > 0$ trace simple closed curves (loops) in the xy-plane. Let's see why.

(a) Divide the second equation in (1) by the first and solve the resulting equation obtaining $y = y(x)$ implicitly; in doing so assume that $x, y > 0$ (so only the first quadrant is considered).

(b) Fix the constant C in your general solution (this gives a specific solution of the equation in (a)). Show that the values of $x, y > 0$ that satisfy the equation lie in the interval $[m_C, M_C]$ for some $m_C, M_C > 0$. Furthermore, for each fixed $x > 0$ at most two values of $y > 0$ satisfy the equation; for each fixed $y > 0$ at most two values of $x > 0$ satisfy the equation.

Hint: Your general solution in (a) should be of the form $G(y) = CF(x)$. Show that $F = F(x)$ and $G = G(y)$ have precisely one critical point in the interval $(0, \infty)$, which is a minimum in one case and a maximum in the other case. Sketch their graphs with x and y both on the horizontal axis. When do they values in common?

(c) Assuming $x, y > 0$, show that $(x'(t), y'(t)) = 0$ if and only if $(x(t), y(t)) = (c/d, a/b)$.

(d) Show that every phase trajectory of (1) in the first quadrant of the xy-plane other than the equilibrium point $(c/d, a/b)$ repeatedly traces a closed curve enclosing $(c/d, a/b)$ in the counter-clockwise direction.