MAT 127: Calculus C, Spring 2022
Homework Assignment 3

WebAssign Problems due before 9am, Wednesday, 02/16
20% bonus for submissions **before 9am, Saturday, 02/12**

Written Assignment due before 4pm, Wednesday, 02/16
in your instructor's office (L01 in Math 44-101B, L02/3 in Math 3-111)

Please read Section 4.3 thoroughly before starting on the problem set; looking over Section 7.3 of the WebAssign textbook may be helpful too.

Written Assignment: Problem C (next page)
Show your work; correct answers without explanation will receive no credit, unless noted otherwise.

Please write your solutions legibly; the graders will disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name (first name first), lecture number (L01, L02, or L03), and HW number in the upper-right corner of the first page.

Problem C

A ball of mass m is projected vertically upward from the earth's surface with a positive velocity v_0. The forces acting on the ball are the force of gravity and the air resistance; the magnitude of the latter is proportional to the speed (the magnitude of the velocity). Thus, by Newton's Second Law, the equation of motion is

$$mv' = ma = -mg - pv,$$

where g and p are positive constants.

(a) Show that the upward velocity of $v = v(t)$, until the ball returns to the ground, is given by

$$v(t) = \left(v_0 + \frac{mg}{p}\right)\mathrm{e}^{-pt/m} - \frac{mg}{p}\,.$$

(b) Show that the height $y = y(t)$ of the ball, until it hits the ground, is given by

$$y(t) = \left(v_0 + \frac{mg}{p}\right)\frac{m}{p}\left(1 - \mathrm{e}^{-pt/m}\right) - \frac{mg}{p}t\,.$$

(c) Show that the amount of time the ball takes to reach the maximum height is

$$t_1 = \frac{m}{p}\ln\left(\frac{mg + pv_0}{mg}\right).$$

Find this time if the mass of the ball is 1 kg, the initial speed is 20 m/s, and the air resistance is .1 kg/s.

(d) Show that

$$y(2t_1) = \frac{m^2 g}{p^2}\left(x - \frac{1}{x} - 2\ln x\right)$$

where $x = \mathrm{e}^{pt_1/m}$ and that for $x > 1$ the function

$$f(x) = x - \frac{1}{x} - 2\ln x$$

is increasing. Use this result to decide whether $y(2t_1)$ is positive or negative. What can you conclude from this? Does the ascent or descent take longer?

(e) What is the answer to the last question in (d) if $p = 0$ (no air resistance) and why? Justify your answer.

(f) Is your answer in (e) consistent with the formula for $y(2t_1)$ in (d) and why?

Hint: take the limit of this formula as $p \longrightarrow 0$.