MAT 127: Calculus C, Spring 2022
Homework Assignment 10

WebAssign Problems due before 9am, Wednesday, 04/20
20% bonus for submissions **before 9am, Saturday, 04/16**

Written Assignment due before 4pm, Wednesday, 04/20
in your instructor's office (L01 in Math 4-101B, L02/3 in Math 3-111)

Please read Sections 6.1,6.2.1/2/4 thoroughly before starting on the problem set; looking over Sections 8.5 and 8.6 of the WebAssign textbook may be helpful too.

Written Assignment: Problems X.1-6,I (below and next page)
Show your work; correct answers without explanation will receive no credit, unless noted otherwise.

Please write your solutions legibly; the graders will disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name (first name first), lecture number (L01, L02, or L03), and HW number in the upper-right corner of the first page.

Problem X.1

Determine the radius and interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 x^n}{2 \cdot 4 \cdot \ldots \cdot 2n}$.

Problem X.2

Suppose the series $\displaystyle\sum_{n=0}^{\infty} c_n x^n$ converges for $x=-4$ and diverges for $x=6$. Do the following series converge or diverge?

$$\text{(a)}\ \sum_{n=0}^{\infty} c_n\,, \qquad \text{(b)}\ \sum_{n=0}^{\infty} c_n 8^n\,, \qquad \text{(c)}\ \sum_{n=0}^{\infty} c_n(-3)^n\,, \qquad \text{(a)}\ \sum_{n=0}^{\infty} (-1)^n c_n 9^n\,.$$

Problem X.3

Find the interval of convergence of the series

$$f(x) = \sum_{n=0}^{\infty} c_n x^n, \qquad \text{where} \quad c_n = \begin{cases} 1, & \text{if } n \text{ is even,} \\ 2, & \text{if } n \text{ is odd} \end{cases}$$

and an explicit formula for $f(x)$.

Problem X.4

Suppose the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$ is R. What is the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^{2n}$?

Problem X.5

Find power series representation for the function $f(x) = \left(\frac{x}{2-x}\right)^3$ and its interval of convergence.

Problem X.6

Let $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2}$. Find the interval of convergence for f, f', f''.

Problem I

(a) Show that the series

$$g(z) = \sum_{n=1}^{\infty} \left(\frac{1}{z-n\pi} + \frac{1}{z+n\pi} \right)$$

converges for every $z \neq m\pi$ for any nonzero integer m and that $g(0)=0$.
Hint: combine the fractions and use the *Absolute Convergence Test*.

(b) The function

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} \left(\frac{1}{z-n\pi} + \frac{1}{z+n\pi} \right)$$

is thus well-defined for every $z \neq m\pi$ for any integer m. Show that

$$\lim_{z \longrightarrow 0} zf(z) = 1, \qquad f(-z) = -f(z), \qquad f(z+\pi) = f(z), \qquad f(\pi/2) = 0, \tag{1}$$

with the middle identities holding whenever either side is defined ($z \neq m\pi$ for any integer m).
Hint: use partial sums for the third equality; the other three are easy.

(c) What is the "simplest" function that satisfies all identities in (1)? (answer only)

Note: This all leads to $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$; see the solutions for more details.