MAT 127: Calculus C, Spring 2022
Homework Assignment 1

WebAssign Problems due before 9am, Wednesday, 02/02
20% bonus for submissions **before 9am, Saturday, 01/29**

Written Assignment due before 4pm, Wendesday, 02/02
in your instructor's office

Please read Section 4.1 thoroughly before starting on the problem set; looking over Section 7.1 of the WebAssign textbook may be helpful too.

Written Assignment: 1.1,1.2,A (below and next page)
Show your work; correct answers without explanation will receive no credit, unless noted otherwise

Please write your solutions legibly; the graders will disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name (first name first), lecture number (L01, L02, or L03), and HW number in the upper-right corner of the first page.

Problem 1.1

Verify that the function $f(t) = -t\cos t - t$ is a solution of the initial-value problem

$$t\,\frac{\mathrm{d}y}{\mathrm{d}t} = y + t^2\sin(t), \qquad y(\pi) = 0.$$

Problem 1.2

The function with the given graph is a solution of one of the following differential equations:

$$\text{(A) } y' = 1 + xy, \qquad \text{(B) } y' = -2xy, \qquad \text{(C) } y' = 1 - 2xy.$$

Decide which is the correct equation and justify your answer.

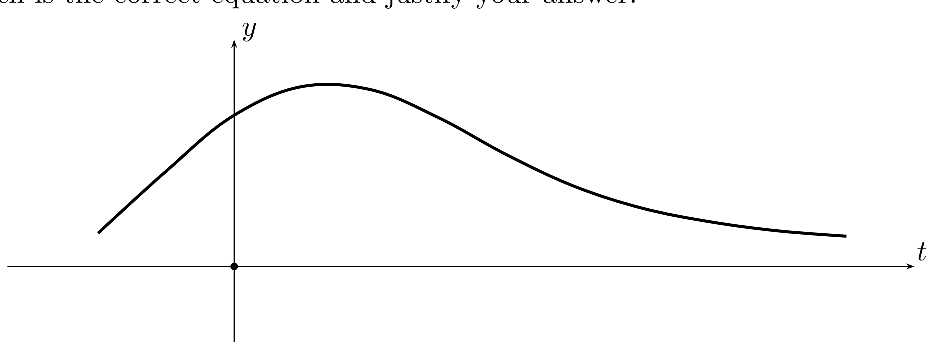

Figure 1: Graph for Problem 1.2

Problem A

(a) State the two Fundamental Theorems of Calculus (answer only).
(b) State the chain rule for one-variable differentiation (answer only).
(c) State the product rule for one-variable differentiation (answer only).
(d) If a is a real number and $f(x)=x^a$, what is $f'(x)$? (answer only)
(e) If $f(x)=e^x$, what is $f'(x)$? (answer only)
(f) State the quotient rule for one-variable differentiation. Deduce it from (b)-(d).
(g) State the change-of-variables formula for one-variable integration. Deduce it from (a) and (b).
(h) State the integration-by-parts formula for one-variable integration. Deduce it from (a) and (c).
(i) Suppose $a=a(t)$ is a smooth function, c is a real number,

$$f(t) = \int_c^t a(s)\mathrm{d}s, \qquad \text{and} \qquad h(t)=e^{f(t)}.$$

Using (a), (b), and (e), show that the function $h = h(t)$ satisfies the differential equation $h' = ah$.