MAT 127: Calculus C, Spring 2022
Final Information

Wednesday, 05/11, 8:15-10:45am, Harriman 137

General Information

(1) Please *bring your Stony Brook ID card to the exam* and show up no later than 8:10am. The exam will begin at 8:15am and you will not receive extra time if you show up after 8:10am.

(2) Please take every other seat starting with the front row. Once a row fills up, please take a seat *directly* behind another person (not diagonally from another person). You can put your bag and/or jacket on one of the seats next to you in the same row.

(3) You will receive an exam booklet (15 pages stapled together), with questions and plenty of space for solutions. Scrap paper will be available upon request. You can staple additional sheets to your exam booklet, but if you do so, please write your name and ID number on each additional sheet and indicate in the exam booklet where to find your solution. Any scrap paper that you do not want to be graded should not be handed in (except separately from the exams, for recycling). The exact front cover of the exam is at the end of this handout; if you have any questions about the instructions, please ask your instructor before the exam.

(4) No notes, books, calculators, or cell phones may be used during the exam. Please bring pencils/pens and an eraser. The *only* items that may be on your desk are pencils/pens, an eraser, exam booklet, and the scrap paper provided by the proctors.

(5) When you receive the exam, please do not open it until the proctors say it is time to start. However, please do fill in your name and Stony Brook ID number and circle your section number on the front cover of the exam.

(6) All problems on the exam should be stated unambiguously. If you feel there is an issue with a statement of a particular problem, please let a proctor know. However, the proctor will not confirm whether your interpretation of the problem is correct.

(7) When you are finished with the exam or when the time is called (whichever comes first), please take your exam booklet to the front along with your Stony Brook ID card. Put the booklet in the pile for your section and sign the photo roster under your picture immediately after. You can leave before the time is over, but please do so as quietly as possible and close the door very gently.

(8) Out of fairness to others, please do not open your exam booklet ahead of time and stop working when the time is over. Your exam score will be reduced by 5 points per minute of either violation.

(9) Copying answers from someone else or allowing someone else to copy your answers would constitute a major breach of the *University Student Conduct Code* and lead to very sad consequences. In particular, you would receive a 0 for the exam and be reported to the Academic Judiciary (which would likely lead to significantly more unpleasant consequences).

Before Final Exam

The grades for the second midterm and HW5-11 must be sorted out by Friday, 5/6; the grades for the first midterm and HW1-4 are no longer subject to change (as stated in *Midterm II Information*). The graders will have more detailed instructions than usual for grading HW12, which should eliminate any potential issues with its grading that may effect your letter grade for the semester.

The final exam will cover essentially all of the course: Chapters 4-6 of the online textbook, Section 7.6 from Stewart's textbook, and *Notes on Second-Order Differential Equations* (the last two are available on the course website). You do not need to remember any of the Taylor series formulas from Sections 6.3 and 6.4 (the relevant ones will be provided for the exam). You should re-read the relevant sections of the textbook and the *ODE Notes* thoroughly, review *Course Summary I-III*, and study the solutions to HW1-12 (even if you did all/most problems correctly). Solutions to HW12 and mini-quizzes will be posted on Thursday, 5/5. Make sure you can do all problem set exercises and and other related problems posted on the course website and in the textbook. *It is far better to be able to do the exercises correctly than to memorize the three chapters of the book or the three course summaries.*

The final exams from Fall 09 and 10 are available on the course website, along with the solutions. Your final exam will be similar topics-wise, but will have 12 problems (with some shorter and answer-only problems replacing some longer ones). Since the final exam is cumulative, it might be good to make sure that you can also do the 2 first midterms and 2 second midterms posted on the course website. For a detailed description of problems that may appear on your exam, see *Types of Problems to Expect* below.

If you have any questions, please come to office hours, MLC, and/or a Residential Tutoring Center. These will run on the regular schedule during the last week of classes (May 2-6). The help hours just before the final exam are

Monday, 5/9	Math Learning Center	11am-2pm in Math S-235, 9-11am&2-8pm on zoom
	Office Hours (Matthew)	1-3pm in Math 4-101B
	Residential Tutoring	evening
Tuesday, 5/9	Math Learning Center	11am-2pm in Math S-235, 9-11am&2-8pm on zoom
	Office Hours (Aleksey)	4-6pm in Math 3-111
	Residential Tutoring	evening

You are welcome to come to any of the above, whether it is run by your instructor or not.

After Final Exam

Detailed solutions to the final exam will be available on the course website on Wednesday, 5/11, evening. Your score for the final exam should appear on *BlackBoard* by Wednesday night as well; this score will be out of 150 (*not* 100). Letter grade breakdowns for the final exam and for the semester are also likely to be posted on Wednesday night. The letter grades for the semester will not be submitted to solar until Thursday, but you'll be able to figure out yours from your *WeiTot* (Weighted Total) on BB and the letter grade breakdowns on Wednesday night. In the highly unlikely case that you qualify for the exception described in *General Course Information*, you'll receive an e-mail on Wednesday night stating so.

BB's computation of WeiTot will be checked for a couple of randomly selected students in each section to make sure that it is programmed correctly. However, you are also encouraged to check that BB computed your WeiTot correctly. In order to do so,

(1) divide each of your homework scores, HW1-HW12, by the maximum possible score for the corresponding HW and multiply by 100;

(2) drop the lowest of the 12 numbers you obtain in this way (if the two lowest scores are the same, drop only one of them; a missing score is same as 0), add up the rest, and divide the total by 11; the result should be the *TotHW* score (this is out of 100);

(3) your WeiTot should be $.22{*}MT1{+}.22{*}MT2{+}.4{*}(FE/1.5){+}.16{*}TotHW$.

If there is a discrepancy, please stop by Thursday's OHs (see below) with a printout of your scores on BB and your computation.

Your letter grade for the semester will be determined **exclusively** by your WeiTot, with the minor exception stated in *General Course Information*; your personal circumstances will not be taken into consideration. Please do not e-mail asking

- whether you can make up any work to bring your WeiTot to the next letter grade cutoff, or
- to give you a higher letter grade than determined by your WeiTot because of your personal circumstances or because your WeiTot falls very close to the next cutoff. You may reduce your chances of putting yourself in the latter situation by doing as well as possible on the remaining HW and by double-checking your work on the final if you finish early (instead of leaving early).

The grading policy in this course applies to everyone in the same way, and it would be inappropriate for this to be otherwise.

While you can review your final exam, it is **not to be removed** from Math 3-111. You can stop by Math 3-111 on Thursday, May 12, 1-3pm, to review your final exam and to pick up your HW12. In order to view your final exam, you must bring a printout of the exam solutions from the course website with you (if you come with a friend, one copy of the solutions will do). This requirement will be waived *only* if you get an A (not even A-) for the course (you'll know from your WeiTot and the posted grade breakdowns).

If you are unable to come to the above end-of-term office hours, you can stop by your instructor's office at the beginning of the Fall term to see your exam. You must still bring a printout of the solutions to the exam with you in order to do so. At that time, you'll be able to pick up your graded HW12, as well as any other problem sets and midterms you have not yet picked up. The grades for the midterms and HW1-11 will no longer be subject to change. If you'd like to discuss how your HW12 was graded with your instructor, you must have solutions to HW12 with you. Any change in your HW12 grade is unlikely to be significant enough to effect the semester grade however.

If you did well in MAT 127, you should be ready for such courses as MAT 203/307, MAT 303, and possibly MAT 319. Some aspects of 203 and 211 are useful for some aspects of 303, but these courses are not officially required for 303 (and 303 typically has higher enrollment than 203, because 303 is more widely applicable). Content-wise, 203/307 is more of a continuation of 125/126, while 303 of 127 (especially of Part I). However, 127/132 is required even for 203/307 as a transition from the lower level of 125/126. MAT 211 is independent of 127, and you may not find it any harder (in fact, 211 is Placement Level 7, while 127 is PL 8, and passing 127 is roughly equivalent to PL 9). If you are intending to take another math course, in particular 203/303, you are likely to have an easier time doing so as soon as possible (i.e. next semester or during the summer). If you'd like to discuss any of the above courses, the office hours on Thursday, 5/12, would be a good time to do this.

Types of Problems to Expect

The final exam will have 12 problems, but one of them will have two versions for you to choose from. Most problems will be worth 10 points each, but some will be worth 15 or 20 points. Most problems will be sub-divided into parts of specified weight. Four standard Taylor series will be provided at the top of the first page of the exam (see the last page of this handout); you can use them (and their intervals of convergence) as appropriate. You must justify any other power series expansion you use, even if it has appeared in class, in the textbook, or on the homework. For example, you might have seen that

$$\frac{x}{(1-x)^2} = \sum_{n=1}^{\infty} nx^n\,.$$

However, if you use this formula on the exam, in order to receive full credit you must justify it. This may mean deriving it from one of the standard formulas provided or directly from the function by computing its Taylor coefficients; quoting the book is not a justification in this case.

The list below should fairly accurately describe the problems that will appear on the exam. Euler's method, the phase-plane (pictures) portion of Stewart's Section 7.6, and your knowledge of the binomial formula will not be directly tested. The items below are listed roughly in the order they have appeared in the course, which is not the order in which they will appear on the exam. In particular, the Part I problems (items (1)-(4) below) will appear after the Part II problems because one of the former will have two versions and will be the last problem for this reason. Questions of types (6) and (7) will form two parts of the same problem. The problems on your exam will be similar in style to the WebAssign and numbered homework problems and the exercises in the textbook, not to the letter homework problems. However, understanding solutions to the letter problems might be helpful (understanding solutions to the other problems is absolutely essential). The hardest topics are perhaps (8) and (12); they are discussed in *Course Summary III*.

(1) Fundamental concepts in first-order differential equations. What does it mean for a given function to be a solution to a given first- or second-order differential equation or initial-value problem? Qualitative behavior of solutions of first-order differential equations (constant solutions, increasing/decreasing, graphs). Direction fields and solution curves. Special properties of direction

fields and solutions of autonomous equations. Related examples: p362/3 1-47; p377-9 66-93; MIf09 1a,3,5b; MIs22 1,5; FEf09 7;FEf10 8.

(2) Separable equations and their applications. This problem may be of one of the following types:

- curve of specified slopes through specified point (including a sketch);
- mixing problem (with constant volume);
- exponential growth/decay equation;
- logistic growth equation.

Because 20 points is a lot to lose for not being able to set up a differential equation modeling a specific problem or a solution to such an equation, this problem will have at least two parts so that it is possible to get credit for later part(s) without the first part. This problem will have two versions, 12A and 12B (on different pages), which will involve two of the above four kinds of problems; your score for Problem 12 will be the higher of your scores on 12A and 12B. While there will be no grading penalty for attempting both versions of the problem, there will be no bonus for this either; so your time might be better spent on getting one of the two versions and the rest of the exam done right, instead of working on both versions. If you finish early, you should double-check that you solved the differential equation or the initial-value problem correctly by plugging your function into the differential equation and also checking that the initial condition is satisfied (if there is one). Make sure your final answer includes the correct physical units if appropriate (this may be appropriate in the last three types of problems, but not in the first two). All four types of problems are discussed in *Course Summary I*, but it is more important to be able to do the exercises correctly. Examples: p241 348-369; p391/2 119-142,148-167; p405 168-188; MIf09 2; MIs22s 3,5; FEf09 8A/B; FEf10 10A/B.

(3) Second-order differential equations. You will be given one or more second-order linear homogeneous differential equations with constant coefficients or initial-value problems involving such equations. In each case, you will be asked to find either the general solution or a solution with specified initial conditions. Your answer should be in the real form (if the roots are complex). If you are asked to find the general solution, *all* you need to do is to write down the corresponding quadratic equation (be careful if some coefficients are 0 though!), find its roots, and use the roots to write down the real form of the general solution depending on whether the two roots are real and distinct, real and the same, or complex conjugates of each other (see *Course Summary I*). Your final answer will involve two constants, C_1 and C_2, in either of the three cases. If you are asked to solve an initial-value problem, you will also need to determine the two constants C_1 and C_2 using the initial values of y and y'; this involves solving a system of two linear equations in C_1 and C_2. Please test your readiness for the general-solution version of this problem by sitting down completely by yourself and writing down the general solutions to the 16 Notes problems (ignore initial conditions if there are any). This should take you 30-40 minutes, after which please compare your answers with someone else in the class (and/or check them by plugging your solution back into the equation). Please test your readiness for the initial-value version of this problem by sitting down completely by yourself and writing down the solutions to Notes Problems 11-16. This should take you 60-80 minutes, after which please compare your answers with someone else in the class (and/or check them by plugging your solution back into the equation and the initial conditions). Examples: Notes 1-16, MIf09 1; MIs22 3; FEf09 6; FEf10 7.

(4) Systems of 2 autonomous first-order differential equations and two-species interactions. Given such a system, you should be able to determine what type of interactions it is modeling and/or which

species corresponds to each of the two variables. You should be able to find the equilibrium points or constant solutions of such a system and explain their significance. Please practice for this problem by finding the equilibrium solutions in all of the systems in the examples below and checking your answers with someone (there are only a few systems there). You do not need to remember the explicit solutions to the exponential growth/decay equation and the logistic equation. However, you must be able to recognize these equations and know what happens to their solutions as the independent variable $t \longrightarrow \infty$; in particular, you need to know what the equilibrium points are. Examples: Stewart's 7.6 1-12; MIIf09 5; MIIs22 5ab; FEf10 9.

(5) Convergence and divergence of sequences and series. You will be given 5 sequences and/or series in this problem and asked to determine whether each of them converges or not, answering by clearly circling YES or NO (but not both). Each correct *answer* will be worth 2 points; no explanation will be expected or considered for grading purposes. If you cannot quickly determine the answer to a question in this problem, leave it until the end, since each question is worth only 2 points of 150. However, do not forget to come back to it and circle one of the possible answers, since there is no penalty for wrong guesses on this problem. This will be Problem 1 on the exam. Please test your readiness for the sequences parts of this problem by sitting down completely by yourself and writing yes/no (for converge/diverge) for each sequence in p493/4 194-228 (keeping in mind why you think so). This should take you 15-20 minutes, after which please compare your answers with someone else in the class. Please test your readiness for the series parts of this problem by sitting down completely by yourself and writing yes/no (for converge/diverge) for each series in p493/4 194-228, p522/3 317-363 (keeping in mind why you think so). This should take you 60-90 minutes, after which please compare your answers with someone else in the class. This problem is meant to be conceptual, testing whether you can tell convergence by the appearance of a sequence or series, not whether you can properly word a formal justification; see *Course Summary II,III*. Examples: HW7 VII.5, HW8 VIII.1, p446 87-92, p482 152-167, p493/4 194-228, p522/3 317-363; MIIf09 1,3; FEf09 1; FEf10 1; (just determine convergence/divergence in all problems).

(6) Computation of limit of a convergent sequence. You will be given a convergent sequence and asked to find its limit. The sequence may be defined by an explicit formula or recursively. As with MIIs22 1abc, this question will be *answer only*; you'll need to put your answer in the box provided. If the box contains anything other than the correct answer, you'll receive no credit for the problem. If the answer is correct, but not in the simplest possible form, you may receive some partial credit depending on the situation (answers like $(\ln 8)/(\ln 2)$ will receive *no* credit even if correct). Once you come up with an answer, try to see if it makes sense by writing out the first few terms of the sequence. For example, if the sequence begins with 5 and seems to be increasing, it should not converge to 0. Examples: p448 27-30,38-43,46-53; MIIs22 1abc; FEf10 2b (compute limits for convergent sequences only).

(7) Expressing a repeating decimal as a simple fraction. Similarly to MIIs22 1d, this question will be *answer only*. Your answer must be written in the box provided in the simplest possible form (e.g. $2/4 = 1/2$). If the box contains anything other than the correct answer, you'll receive no credit for this question; if the answer is correct, but not in the simplest possible form, you'll receive partial credit. Since you must get the answer completely right in order to receive any credit for this question (which is fairly easy), make sure to do lots of practice questions of this form in order to avoid computational errors. In particular, please do all of the examples below and make up your own (this is easy to do in this case). You can check your answers for these practice problems by using a calculator; on the exam, you can check your answer by doing long

division on paper. If you did not get 4 or 5 points for MIIs22 1d, this was likely due to a simple arithmetic mistake; most likely, not simplifying the fractions as early as possible contributed to making such a mistake (it is easier to add 1 to 3/55 than to 54/990). Once you get an answer, check it directly if you have time left. While some answers for MIIs22 1d were only slightly off and would have required long division to catch this, a number of answers were way off, being either clearly smaller than 1 or greater than 2. Even doing long division in MIIs22 1d would not have been so bad: to divide by 55, multiply by 2, then divide by 11, and shift the decimal point one digit to the right. Examples: HW7 WA 4,5; MIIf09 4a; MIIs22 1d; FEf10 2a.

(8) Estimating sums of series. You will be given a series and asked to estimate its sum with specified precision by using the minimal possible number of terms. The latter is done by choosing the smallest integer m such that the sum of the series starting with the $(m+1)$-th term is no larger than the specified precision and then summing the first m terms of the series to get the estimate. You'll need to justify why the integer m you choose provides an estimate for the series with the required precision. Based on what you have learned in class, you should know how to choose the required integer m (which is the substance of such problems) only for two types of series $\sum_{n=1}^{\infty} a_n$:

- $a_n = f(n)$ for a positive, continuous, decreasing to 0 function $f = f(x)$ defined for $x \geq 1$;
- $a_n \longrightarrow 0$, $|a_n| > |a_{n+1}|$, and the signs of terms a_n strictly alternate between $+$ and $-$.

In both cases, the estimates are related to certain convergence tests for series. Make sure to check that the assumptions required for the two tests are satisfied (state the relevant properties in your setting and justify them if necessary); each of the two tests has three assumptions. The correct value of m will be sufficiently small so that you can add up the first m terms and leave your answer as a simple fraction (p/q for some integers p and q with no common factor). You should also be able to tell whether your estimate is an under- or over-estimate for the sum. Examples: p483 173-182; p506 280-285; HW8 WA 1; HW9 WA 2, IX.2; FEf09 5; FEf10 6.

(9) Power series, limits, and integration. Given a power series, you should be able to determine its radius and interval of convergence, find a limit involving the function defined by this power series, and find an anti-derivative of a related function as a power series, along with its radius and interval of convergence. Examples: p541/2 7-42; p559 87-90; p580 170-173; HW11 WA 1-4; FEf09 3; FEf10 4.

(10) Conceptual questions on sequences, infinite series, and power series. The meaning and qualitative properties of the interval of convergence. Applications of convergence tests for infinite series with abstract coefficients. The spirit of this problem will be similar to MIIs22 2, but you will not need to justify your answers. Examples: p541 1-6; HW10 X.2; MIIs22 2.

(11) Taylor series, radius and interval of convergence of power series. Given a function, find its Taylor series around the specified center and determine the radius and interval of convergence of the resulting power series. Taylor series for four standard functions will be provided on the second page of the exam; one or more of them may be useful in some of the cases. Examples: p559 95-101,104-107; p578/9 116-123, 140-159; HW10 WA 3-8; FEf09 2; FEf10 3.

(12) Convergence of series and computation of their sums via Taylor series. Given a convergent infinite series, explain why it converges and find its sum. These two things can be done independently of each other. In order to compute the sum, you will need to represent the infinite series as the Taylor series of some function $f = f(x)$ evaluated at some specific value of x (if you find a

different way of obtaining the sum, that is fine). The provided standard series might again be useful; you may need to differentiate, integrate or multiply them by a power of x though. You can justify the convergence of the infinite series either by showing that the evaluation point lies in the interval of convergence of the power series or by using one or more of the many convergence tests in 5.3-5.6. In the latter approach, the Ratio Test is likely to be your best bet, as it usually works best with power series. As was the case on Midterm II, you do not have to memorize the names of the tests, but you have to make it clear what test you are using and check the required assumptions (see *Midterm II Info* for examples). Examples: p559 91-94; p579 160-163; HW11 WA 5-9, XI.1; FEf09 4; FEf10 5.

MAT 127 Final Exam

May 11, 2022 8:15-10:45am

Name:____________________________ **ID:**__________________

first name *first*

Section:	L01	L02	L03	(circle yours)
	MW 4:25-5:45pm	TuTh 9:45-11:05am	TuTh 1:15-2:35pm	

DOO NOT OPEN THIS EXAM YET

Instructions

(1) Fill in your name and Stony Brook ID number and circle your lecture number at the top of this cover sheet.

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Please write legibly to receive credit. Circle or box your final answers. If your solution to a problem does not fit on the page on which the problem is stated, please indicate on that page where in the exam to find (the rest of) your solution.

(4) You may continue your solutions on additional sheets of paper provided by the proctors. If you do so, please write your name and ID number at the top of each of them and staple them to the back of the exam (stapler available); otherwise, these sheets may get lost.

(5) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.

(6) Leave your answers in exact form (e.g. $\sqrt{2}$, not ≈ 1.4) and simplify them as much as possible (e.g. $1/2$, not $2/4$) to receive full credit.

(7) Show your work; correct answers only will receive only partial credit (unless noted otherwise).

(8) Be careful to avoid making grievous errors that are subject to heavy penalties.

(9) If you need more blank paper, ask a proctor.

Out of fairness to others, please **stop working and close the exam as soon as the time is called**. A significant number of points will be taken off your exam score if you continue working after the time is called. You will be given a two-minute warning before the end.

Some Taylor Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{if} \quad |x| < 1$$

$$\mathrm{e}^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

to receive full credit, justify any other power series expansion you use