# MAT 127: Calculus C, Spring 2012
## Midterm II Information

**Thursday, 04/07, 8:15-9:35pm, Engineering 143**

### General Information

(1) *You must bring your Stony Brook ID card to the exam.*

(2) Please show up no later than 8:10pm. The exam will begin at 8:15pm and you will not receive extra time if you show up after 8:10pm.

(3) Please take every other seat starting with the front row. Once a row fills up, please take a seat *directly* behind another person (not diagonally from another person). You can put your bag and/or jacket on one of the seats next to you in the same row.

(4) You will receive an exam booklet (6 pages stapled together), with questions and plenty of space for solutions. Scrap paper will be available upon request. You can staple additional sheets to your exam booklet, but if you do so, please write your name and ID number on each additional sheet and indicate in the exam booklet where to find your solution. Any scrap paper that you do not want to be graded should not be handed in (except separately from the exams, for recycling). The exact front cover of the exam (except for the grade box) is at the end of this handout; if you have any questions about the instructions, please ask your instructor before the exam.

(5) No notes, books, calculators, or cell phones may be used during the exam. Please bring pencils/pens and an eraser. The *only* items that may be on your desk are pencils/pens, an eraser, exam booklet, and the scrap paper provided by the proctors.

(6) When you receive the exam, please do not open it until the proctors say it is time to start. However, please do fill in your name and Stony Brook ID number and circle your section number on the front cover of the exam.

(7) All problems on the exam should be stated unambiguously. If you feel there is an issue with a statement of a particular problem, please let a proctor know; however, the proctor will not confirm whether your interpretation of the problem is correct.

(8) When you are finished with the exam or when the time is called (whichever comes first), please take your exam booklet to the front along with your Stony Brook ID card. Put the booklet in the pile for your section and sign the photo roster under your picture immediately after. You can leave before the time is over, but please do so as quietly as possible and close the door very gently.

(9) Out of fairness to others, please do not open your exam booklet ahead of time and stop working when the time is over. Your exam score will be reduced by 5 points per minute of either violation.

(10) Copying answers from someone else or allowing someone else to copy your answers would constitute a major breach of the *University Student Conduct Code* and lead to very sad consequences. In particular, you would receive a 0 for the exam and be reported to the Academic Judiciary (which would likely lead to significantly more unpleasant consequences).

**Before Midterm II**

The midterm will cover Sections 7.6 of Stewart's textbook (posted on the website), and the *Notes on the Ratio Test*, Chapter 5 of the online textbook except for the *Alternating Series Test* and *Estimating the Sum of a Series* (via the Integral Test in Section 5.3 and the Alternating Series Test in Section 5.5). You should re-read these sections thoroughly, review *Course Summary II*, and study the solutions to PS5-8 (even if you did all/most problems correctly). Make sure you can do all problem set exercises from the above sections. As the second midterm is based on only 4 problem sets, skipping any one of them would drastically reduce your chances of passing it.

The second midterm from Fall 09 are available on the course website, along with solutions. Please try doing it in 90 minutes before looking at the solutions under full test conditions (no calculators, no solutions, no distractions). If you do not do well on these midterms, you should take this as a major warning. The second midterm in this class will be similar in many aspects to these midterms, though there will be some differences.

The grades in MAT 127 have had a long history of dropping significantly from Midterm I to Midterm II. In Fall 09, drops of around 15 points (out of 100) were typical, with a few gains of a few points and a number of 30-40 point drops. Most of the 30-40 points drops are likely to come from those of you who scored in the A/A- range on the first midterm, but have since gotten a bit overly confident perhaps. Fortunately, such a huge drop from the A/A- range on the first midterm should still leave you with a realistic chance of passing this course (if you catch up before the final exam), but unfortunately with essentially no chance of getting an A/A- for the semester. As this class is not curved, the letter grade cutoffs will not change significantly from the first midterm. As a result, there are likely to be fewer scores in the A/A- range on the second midterm than there were in the 90s on the first midterm. If you do not do reasonably on the second midterm, it will be very hard to compensate for this on the final (which will be cumulative).

Stewart's Section 7.6 is pretty hard being heavily graphics and graphics of rather difficult kind, but at least it is more concrete than Chapter 5 of the online textbook. The latter often lead to confusion between sequences and series, the corresponding notions of convergence, and the corresponding convergence/divergence tests. The only way to void this is by doing **lots of** exercises from the textbook. The reason that the recent problem sets contained so many exercises is that most of them should take you very little time. If you have really mastered Chapter 5, you should be able to tell whether most sequences and series in the exercises for these sections converge or diverge *immediately* and to formally justify your answer in 1-2 minutes (a few of the series there require computing partial sums, which takes a little bit of time). If you want to increase your chances of passing this course after a poor score on the first midterm or want a good grade after a strong score on the first midterm, you should do *every* exercise in these sections that asks to determine whether a sequence or a series converges (this excludes a few exercises, involving long statements or approximating sums of series); you can check your answers at OHs, in MLC, or with other students.

If you received an F or D on the first midterm, you should have moved to MAT 126 or 132 by now. The deadline to withdraw from this class or switch to GPNC is 4pm on Friday, April 1. If you received a C on the first midterm, please do the second midterm from Fall 09 under test conditions before April 1; if you do very poorly on them, you should probably withdraw from the class while you can or at least switch to GPNC. If you withdraw from this class, you will receive a W on your transcript and would need to ask for a retake permission, but if you fail this class, you'll still need to ask for a retake permission and will also hurt your GPA and possibly performance in your other

courses.

If you have any questions, please come to office hours (lots of them on Monday-Thursday!), MLC, and/or a Residential Tutoring Center.

**Note** that any possible issues concerning your grades on Midterm I and HW3,4 must be resolved before Midterm II. Midterm I and HW1-4 grades will not be changed after Midterm II, even if your score was simply tallied incorrectly or mis-recorded in *Blackboard*. The HW1,2 scores are no longer subject to change.

## After Midterm II

Detailed solutions to the midterm will be available on the course website on Friday morning; please print these out before the following lecture. If the solutions do not satisfactorily explain how your solution to a particular problem was graded, please check with the primary grader for the given problem (the primary graders for all problems will be listed on the website). You must bring a printout of the solutions to the exam when you meet with the grader. Your exam grade will be changed, up or down, only if your problem was graded contrary to the grading scheme described in the solutions or inconsistently with others. If your total exam score was incorrectly tallied, please let your instructor know.

## Types of Problems to Expect

The second midterm will have 5 problems, not of equal weight, with some problems sub-divided into questions of specified weight. Your midterm will be similar in many aspects to the second midterm in Fall 09, but there will be some differences. The ability to solve first and second-order equations is not required for the second midterm (but it will be tested again on the final). The list below should fairly accurately describe the problems that will appear on the exam. Items (1)-(6) below are listed in the order they have appeared in the course, which is not necessarily the order in which they will appear on the exam. The problems on your midterm will be similar in style to the WebAssign problems and the numbered HW problems, not to the letter problems on the problem sets. However, understanding solutions to the letter problems might be helpful (understanding solutions to the textbook problems is necessary).

(1) Systems of 2 autonomous first-order differential equations and two-species interactions. Given such a system, you should be able to determine what type of interactions it is modeling and/or which species corresponds to each of the two variables. You should be able to find the equilibrium points or constant solutions of such a system and explain their significance. Please practice for this part of the problem by finding the equilibrium solutions in all of the systems in the examples below and checking your answers with someone (there are only a few systems there and doing this may help you get a few extra points on the midterm). Given a phase trajectory, you'll need to be able to sketch graphs of the corresponding functions and vice versa; in both cases, the axes should have appropriate labels and you need to be able to explain the sketching process. This may require estimating coordinates of some points. Your estimates should be reasonable and consistent. For example, if a coordinate of some point appears to be roughly half way between 200 and 300 and 250 is not marked, anything between 230 and 270 would be reasonable. However, if one point lies to the left of another,

your estimate for the horizontal coordinate of the former should be smaller than for the latter. If you are given that one of the coordinates of a point is 250, you should not change it to 255. You may want to use color pencils and/or pens, but please do not use red or green pens; red or green pencils are ok if they are clearly distinguishable from pens. You do not need to remember the explicit solutions to the exponential growth/decay equation and the logistic equation. However, you must be able to recognize these equations and know what happens to their solutions as the independent variable $t \longrightarrow \infty$; in particular, you need to know what the equilibrium points are. Examples: 7.6 1-12; MIIf09 5

(2) Computation of limits of convergent sequences. You will be given more than one convergent sequences and asked to find their limits. You can reasonably expect something very standard as well as something less standard, in the style of HW6 WA 3. This question will be *answer only*. Your answer to each of these questions must be written in the box provided in the simplest possible form. If the box contains anything other than the correct answer, you'll receive no credit for the problem; if the answer is correct, but not in the simplest possible form, you may receive some partial credit depending on the situation (answers like $(\ln 8)/(\ln 2)$ will receive *no* credit even if correct). Examples: p448 27-30,38-43,46-53

(3) Expressing a repeating decimal as a simple fraction. This question will be *answer only*. Your answer must be written in the box provided in the simplest possible form (e.g. $2/4 = 1/2$). If the box contains anything other than the correct answer, you'll receive no credit for this question; if the answer is correct, but not in the simplest possible form, you'll receive partial credit. Since you must get the answer completely right in order to receive any credit for this question (which is fairly easy), make sure to do lots of practice questions of this form in order to avoid computational errors. In particular, please do all of the examples below and make up your own (this is easy to do in this case). You can check your answers for these practice problems by using a calculator; on the exam, you can check your answer by doing long division on paper. Examples: HW7 WA 4,5; MIIf09 4a

(4) Convergence and divergence of sequences and series. You will be asked how the convergence of related sequences and series is correlated. For example, if a sequence converges/diverges, what can you say about the corresponding series? Examples: HW8 VIII.2,3; p479/80 129-133; p494 235-241

(5) Partial sums and sums of convergent series. In this problem, you will be given some series and asked to find the corresponding sequences of partial sums, to determine whether the series converges, and to find the sum of the series if this is the case. You may need to use the series rules, sum of geometric series, and/or telescoping cancellation possibly along with partial fractions. Examples: HW7 WA 3,7,VII.1,3; p467 101-114; MIIf09 4b

(6) Convergence and divergence of series. You'll be given a series and asked to determine whether it converges or diverges and if so why. You may also be given a series which involves a parameter, in which case you will be asked to find all values of the parameter for which the series converges. In the first case, you'll need to circle your choice of answer to the right of the question; in the second case, you'll need to write your answer in the box provided to the right of the question. In both cases, you will not receive the portion of the credit allocated for the answer if you fail to do so. You'll need to justify your answers in the space provided below each question. In each case, different convergence/divergence tests for series may be usable, but some are likely to be easier to use than others. Examples: HW7 VII.5, HW8 VIII.1, p646 87-92, p482 152-167, p493/4 194-228, p522/3 317-363 MIIf09 3

While you need to know, understand, and be able to use all convergence tests for sequences and series, you do not need to memorize the precise name of each test. However, your argument must it clear what you are using. For example, if you are asked whether the sequence $a_n = 1 + (-1)^n/n$ converges or diverges, you could say:

> $\boxed{\textit{converges}}$ *because* $1 - 1/n \le a_n \le 1 + 1/n$ *and the sequences* $b_n = 1 - 1/n$ *and* $c_n = 1 + 1/n$ *converge to the same limit, which is* 1.

This would receive full credit; what has just been used is the *Squeeze Theorem for Sequences*, but you do not have to state it. By the same theorem, $\lim_{n \longrightarrow \infty} a_n = 1$.

In some cases, there are different ways to justify your answer. For example, suppose you are asked whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges or diverges. You could say

> $\boxed{\textit{converges}}$ *because* $0 \le \dfrac{1}{n^2+1} \le \dfrac{1}{n^2}$ *and* $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ *converges.*

This uses the *Comparison Test* and the *p-Series Test*; the former requires the terms in the series to be nonnegative, and so you should make it clear that you are aware of this. You could instead say

> $\boxed{\textit{converges}}$ *because* $\dfrac{1}{n^2+1}, \dfrac{1}{n^2} > 0$, $\displaystyle\lim_{n \longrightarrow \infty} \frac{n^2+1}{n^2} = \lim_{n \longrightarrow \infty} (1 + 1/n^2) = 1$ *and* $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ *converges.*

This uses the *Limit Comparison Test* and the *p-Series Test*. A third possibility is

> $\boxed{\textit{converges}}$ *because* $f(x) = \dfrac{1}{1+x^2}$ *is a continuous, positive, and decreasing function on* $[1, \infty)$ *and*
>
> $$\int_1^{\infty} \frac{1}{1+x^2} \mathrm{d}x = \arctan x \Big|_{x=1}^{\infty} = \lim_{x \longrightarrow \infty} \arctan x - \arctan 1 = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$
>
> *is finite.*

This uses the *Integral Test*. The last argument requires knowing the integral of $1/(1+x^2)$ and that $\arctan x$ approaches a finite value as $x \longrightarrow \infty$. If properly worded, it does not actually require knowing that this finite value is $\pi/2$ or that that $\arctan 1 = \pi/4$. The latter is avoidable by stating that $\arctan x$ is defined for all $x$. Whenever your answer can be justified in different ways, you should give *only one* correct and complete justification. If you give more than one justification and every one of them is correct and complete, you will receive full credit. However, if one of them contains an error, you will lose points even if another explanation is correct and complete.

# MAT 127 Midterm II

**April 7, 2022 8:15-9:35pm**

**Name:**_____ **ID:**_____

first name *first*

**Section:** L01 L02 L03 (circle yours)

MW 4:25-5:45pm TuTh 9:45-11:05am TuTh 1:15-2:35pm

# DO NOT OPEN THIS EXAM YET

**Instructions**

(1) Fill in your name and Stony Brook ID number and circle your lecture number at the top of this cover sheet.

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Please write legibly to receive credit. Circle or box your final answers. If your solution to a problem does not fit on the page on which the problem is stated, please indicate on that page where in the exam to find (the rest of) your solution.

(4) You may continue your solutions on additional sheets of paper provided by the proctors. If you do so, please write your name and ID number at the top of each of them and staple them to the back of the exam (stapler available); otherwise, these sheets may get lost.

(5) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.

(6) Leave your answers in exact form (e.g. $\sqrt{2}$, not $\approx 1.4$) and simplify them as much as possible (e.g. 1/2, not 2/4) to receive full credit.

(7) Show your work; correct answers only will receive only partial credit (unless noted otherwise).

(8) Be careful to avoid making grievous errors that are subject to heavy penalties.

(9) If you need more blank paper, ask a proctor.

Out of fairness to others, please **stop working and close the exam as soon as the time is called**. A significant number of points will be taken off your exam score if you continue working after the time is called. You will be given a two-minute warning before the end.