MAT 127 Midterm II

November 4, 2009
8:30-10:00pm

Name:____________________________ **ID:**__________________

Section:	L01	L02	L03	L04	(circle yours)
	MWF 9:35-10:30am	MW 5:20-6:45pm	TuTh 2:20-3:40pm	TuTh 5:20-6:40pm	

DO NOT OPEN THIS EXAM YET

Instructions

(1) This exam is closed-book and closed-notes; no calculators, no phones.
(2) Please write legibly. Circle or box your final answers.
(3) Show your work. Correct answers only will receive only partial credit.
(4) Simplify your answers as much as possible.
(5) Leave your answers in exact form (e.g. $\sqrt{2}$, not ≈ 1.4).
(6) If you need more blank paper, ask a proctor.
(7) Please write your name and ID number on any additional sheets you'd like to be graded and **staple** them to the back of the exam (stapler provided); indicate in the exam that the solution continues on the attached sheets.
(8) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.

Out of fairness to others, please **stop working and close the exam as soon as the time is called**. A significant number of points will be taken off your exam score if you continue working after the time is called. You will be given a two-minute warning before the end.

1 (15pts)	
2 (20pts)	
3 (20pts)	
4 (20pts)	
5 (25pts)	
Total (100pts)	

Problem 1 (15pts)

Determine whether each of the following sequences converges or diverges; justify your answer. You do not have to determine the limit if the sequence converges.

(a; 5pts) $a_n = \dfrac{(-1)^n\, n^4}{3n^4 + 1}$

(b; 5pts) $a_n = 1 + \dfrac{\sin(n\pi/2)\ln n}{n}$

(c; 5pts) $a_1 = 1, \quad a_{n+1} = \dfrac{a_n + 9}{2}$

Problem 2 (20pts)

Each of the following sequences converges (you do not need to check this); determine its limit (show your work!).

(a; 6pts) $\quad a_n = \dfrac{n}{1+\sqrt{4n^2+1}}$

(b; 7pts) $\quad a_n = \dfrac{5^n}{n!}$ (reminder: $n! = 1 \cdot 2 \cdot \ldots \cdot n$)

(c; 7pts) $\quad \sqrt{6},\ \sqrt{6+\sqrt{6}},\ \sqrt{6+\sqrt{6+\sqrt{6}}},\ \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}},\ \ldots$

Problem 3 (20pts)

Determine whether each of the following series converges or diverges; justify your answer. You do not have to compute the sum if the series converges.

(a; 6pts) $\displaystyle\sum_{n=1}^{\infty}(-1)^n$

(b; 7pts) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{3^n-2^n}$

(c; 7pts) $\displaystyle\sum_{n=1}^{\infty}\tan^2(1/n)$ (reminder: $\tan\theta = \frac{\sin\theta}{\cos\theta}$)

Problem 4 (20pts)

(a; 10pts) Write the number $2.0\bar{7} = 2.077777\ldots$ as a simple fraction (p/q for some integers p and q).

(b; 10pts) Determine the sum of the following convergent series:

$$\sum_{n=1}^{\infty} \frac{1}{4n^2 - 9} = -\frac{1}{5} + \frac{1}{7} + \frac{1}{27} + \frac{1}{55} + \frac{1}{91} + \ldots$$

Hint: partial fractions

Problem 5 (25pts)

The populations of bears and salmon in Salmon River Valley are modeled by the following system of differential equations

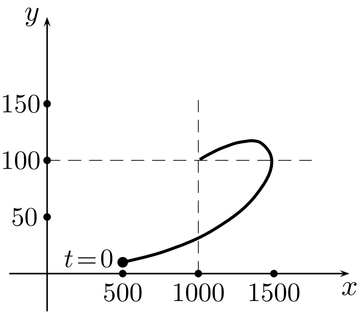
$$\begin{cases} \frac{\mathrm{d}x}{\mathrm{d}t} = x - \frac{1}{100}xy \\ \frac{\mathrm{d}y}{\mathrm{d}t} = y - \frac{1}{50}y^2 + \frac{1}{1000}xy \end{cases} \qquad (x,y) = (x(t), y(t)),$$

where t denotes time. While the bears can live off berries and other plants, they also feed on the salmon in the river. The salmon has no other predators in the valley.

(a; 5pts) Does the function $x = x(t)$ above represent the population of the bears or of the salmon? Explain why.

(b; 10pts) Find all equilibrium (constant) solutions of the system (show your work!) and explain their significance.

(c; 10pts) The left diagram below shows the phase trajectory for the above system of differential equations that starts at (500,10) at $t = 0$. Sketch the corresponding graphs of $x = x(t)$ and $y = y(t)$ as functions of time on the right diagram below; add appropriate markings to the axes. Explain how you make your sketch!



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