MAT 127 Midterm II

April 7, 2022 8:15-9:35pm

Name: _____ **ID:** _____

first name *first*

Section: L01 MW 4:25-5:45pm L02 TuTh 9:45-11:05am L03 TuTh 1:15-2:35pm (circle yours)

DO NOT OPEN THIS EXAM YET

Instructions

(1) Fill in your name and Stony Brook ID number and circle your lecture number at the top of this cover sheet.

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Please write legibly to receive credit. Circle or box your final answers. If your solution to a problem does not fit on the page on which the problem is stated, please indicate on that page where in the exam to find (the rest of) your solution.

(4) You may continue your solutions on additional sheets of paper provided by the proctors. If you do so, please write your name and ID number at the top of each of them and staple them to the back of the exam (stapler available); otherwise, these sheets may get lost.

(5) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.

(6) Leave your answers in exact form (e.g. $\sqrt{2}$, not ≈ 1.4) and simplify them as much as possible (e.g. 1/2, not 2/4) to receive full credit.

(7) Show your work; correct answers only will receive only partial credit (unless noted otherwise).

(8) Be careful to avoid making grievous errors that are subject to heavy penalties.

(9) If you need more blank paper, ask a proctor.

Out of fairness to others, please **stop working and close the exam as soon as the time is called**. A significant number of points will be taken off your exam score if you continue working after the time is called. You will be given a two-minute warning before the end.

1 (20pts)	2 (15pts)	3 (15pts)	4 (10pts)	5 (20pts)	Tot (80pts)

Problem 1 (20pts)

Answer Only: no explanation is required. Write your answer to each question in the corresponding box *in the simplest possible form.* No credit will be awarded if the answer in the box is wrong; partial credit may be awarded if the answer in the box is correct, but not in the simplest possible form. In (a)-(c), assume that the limits exist.

(a; 5pts) Find the limit of the sequence $a_n = \cos\left(\dfrac{n\pi}{n+\pi}\right)$

(b; 5pts) Find the limit of the sequence $a_n = \left(1 - \dfrac{3}{n}\right)^{9n}$

(c; 5pts) Find the limit of the sequence recursively defined by

$$a_1 = 1, \qquad a_{n+1} = \frac{1}{1+a_n} \quad \text{if } n \geq 1$$

(d; 5pts) Write the number $1.0\overline{54} = 1.0545454\ldots$ as a simple fraction

Problem 2 (15pts)

Suppose $a_1, a_2, a_3, \ldots$ is a sequence such that $a_1, a_2, a_3 \ldots \geq 0$ and the series $\sum_{n=2}^{\infty} a_n$ converges. For each question below, circle your answer and justify it below

(a) Does the sequence $a_1, a_2, a_3, \ldots$ converge? **yes** **no** **impossible to tell**

(b) Does the series $\sum_{n=2}^{\infty} \frac{1}{2+a_n}$ converge? **yes** **no** **impossible to tell**

(c) Does the series $\sum_{n=1}^{\infty} \sqrt{a_n}$ converge? **yes** **no** **impossible to tell**

(d) Does the series $\sum_{n=1}^{\infty} a_n^2$ converge? **yes** **no** **impossible to tell**

Problem 3 (15pts)

Find all values of p for which the series

$$\sum_{n=1}^{\infty}\left(\frac{n^p}{n^3+6n^2+11n+6}+\frac{5^{n/2}}{3^{n+p}}\right)$$

converges. Write your answer in the box to the right and justify it below.

(bonus 10pts) Pick any value of p for which the above series converges and find the sum of the resulting series explicitly.

Warning. The bonus part is hard and is subject to harsh grading. Your time is likely to be better spent double- and triple-checking your work on the rest of the exam. If you do not see better ways of using your time on the exam, please answer this question either on the facing page or on the back of this page and state below where to find the answer.

Problem 4 (10pts)

Determine the sequence s_n of partial sums (sum of the first n terms) corresponding to the series

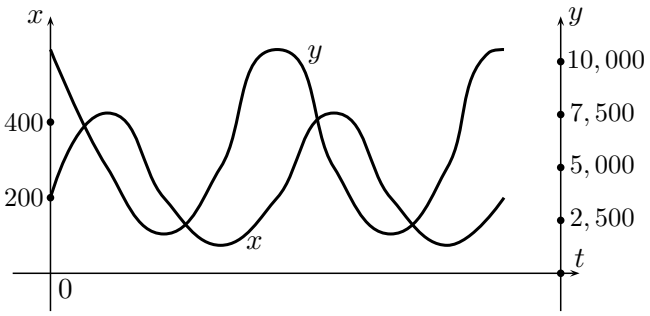
$$\sum_{n=1}^{\infty}(-1)^n\left(\cos\left(\frac{\pi}{2n}\right)-\cos\left(\frac{\pi}{2(n+2)}\right)\right).$$

Does this series converge? If so, what is its sum? Justify your answers.

Problem 5 (20pts)

A two-species interaction is modeled by the system of differential equations below, with t denoting time.

$$\begin{cases} \frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{10}x + \frac{1}{50{,}000}xy \\ \frac{\mathrm{d}y}{\mathrm{d}t} = y - \frac{1}{200}xy \end{cases}$$



(a; 3pts) Which of the following best describes the interaction modeled by this system?

(i) predator-prey (ii) competition for same resources (iii) cooperation for mutual benefit

Circle your answer above and justify it below.

(b; 7pts) Find the equilibrium (constant) solutions of the system and explain their significance relative to the interaction the system is modeling. **Answer Only:** clearly write down each equilibrium solution followed by its significance below, with one of these statements per line. Use scrap paper or the back side of a page in the exam to work out your answer.

(c; 10pts) The diagram above shows the graphs of functions $x = x(t)$ and $y = y(t)$ so that the pair (x, y) solves the above system of differential equations. Sketch the corresponding (directed) phase trajectory below, indicating coordinates of whatever points possible. Explain/indicate how you make your sketch!