MAT 127 Midterm I

October 7, 2009
8:30-10:00pm

Name:______________________________ **ID:**__________________

Section: L01 (MWF 9:35-10:30am) L02 (MW 5:20-6:45pm) L03 (TuTh 2:20-3:40pm) L04 (TuTh 5:20-6:40pm) (circle yours)

DO NOT OPEN THIS EXAM YET

Instructions

(1) This exam is closed-book and closed-notes; no calculators, no phones.
(2) Please write legibly. Circle or box your final answers.
(3) Show your work. Correct answers only will receive only partial credit.
(4) Simplify your answers as much as possible.
(5) Leave your answers in exact form (e.g. $\sqrt{2}$, not ≈ 1.4).
(6) If you need more blank paper, ask a proctor.
(7) Please write your name and ID number on any additional sheets you'd like to be graded and staple them to the back of the exam (stapler provided); indicate in the exam that the solution continues on the attached sheets.
(8) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.

Out of fairness to others, please **stop working and close the exam as soon as the time is called**. A significant number of points will be taken off your exam score if you continue working after the time is called. You will be given a two-minute warning before the end.

1 (20 pts)	
2 (20 pts)	
3 (15 pts)	
4 (20 pts)	
4 (25 pts)	
Total (100pts)	

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Problem 1 (20pts)

(a; 8pts) Show that the function $y(x) = 2e^x$ is a solution to the initial-value problem

$$y'' - 6y' + 5y = 0, \quad y = y(x), \qquad y(0) = 2, \qquad y'(0) = 2.$$

(b; 10pts) Find the general solution $y=y(x)$ to the differential equation

$$y'' - 6y' + 5y = 0, \qquad y = y(x).$$

(c; 2pts) What is the relation between the solution given in (a) and the general solution in (b)?

Problem 2 (20pts)

A bacteria culture grows at a rate proportional to its size. It contained 125 cells at 8pm and 250 cells at 8:20pm.

(a; 12pts) Find an expression for the number of cells in the culture t minutes after 8pm.

(b; 8pts) When will the bacteria culture reach 2,000 cells?

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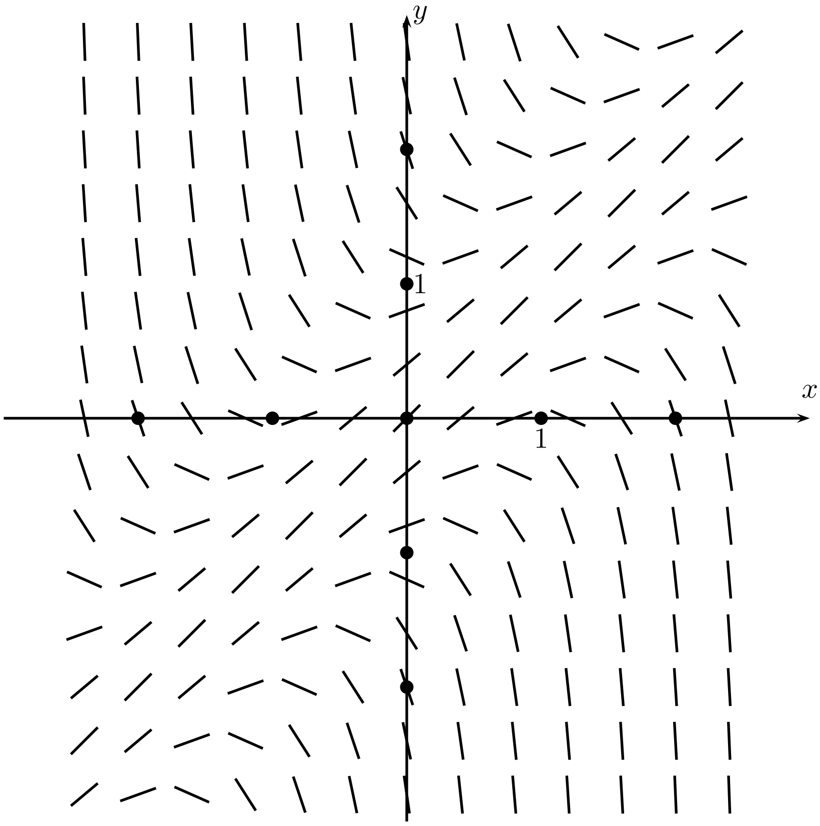
Problem 3 (15pts)

The direction field for a differential equation is shown below.



(a; 10pts) On the direction field, sketch and *clearly label* the graphs of the three solutions with the initial conditions $y(0) = -.8$, $y(0) = 0$, and $y(0) = 2$ (each of these three conditions determines a solution to the differential equation).

(b; 5pts) The direction field above is for one of the following differential equations for $y = y(x)$:

$$\text{(i) } y' = 1 - y^2\,, \qquad \text{(ii) } y' = 1 - (x-y)^2\,, \qquad \text{(iii) } y' = 1 - (x+y)^2\,.$$

Which of these three equations does the direction field correspond to and why?

Problem 4 (20pts)

(a; 15pts) Let $y = y(x)$ be the solution to the initial-value problem

$$y' = xy, \quad y = y(x), \qquad y(0) = 1.$$

Use Euler's method with $n = 3$ steps to estimate the value of $y(1)$. Show your steps clearly and use simple fractions (so $5/4$ or $\frac{5}{4}$, not 1.25).

(b; 5pts) Sketch the path in the xy-plane that represents the approximation carried out in part (a) and indicate its (path's) primary relation to the graph of the actual solution $y = y(x)$ of the initial-value problem in (a).

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Problem 5 (25pts)

Find the general solution $y = y(x)$ to the differential equation

$$y' + 9 = y^2, \qquad y = y(x).$$

Sketch at least five solution curves, on the same plot of the xy-plane, representing every possible type of behavior of the solutions $y = y(x)$ depending on the value of $y(0)$; justify the features exhibited on your plot (this can be done with or without using the general solution). You can add comments to your plot to more clearly identify the features it is meant to exhibit.

you can write on this page