MAT 127: Calculus C, Fall 2009
Homework Assignment 9

Problem Set 9 is due before the beginning of lecture on
Wednesday, 11/18 if enrolled in L01, L02
Thursday, 11/19 if enrolled in L03, L04

This problem set is a bit longer than usual, since it covers about 1.5 weeks.

Please read Section 8.4 and 8.5 thoroughly before starting on the problem set.

Problem Set 9: 8.4 6,8,18,22,29,31,33*,36; 8.5 6,16,18,20,25,28; Problem G (see below)
*show work for all four series; justify your answers on all problems

Please write your solutions legibly; the graders may disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name and lecture number in the upper-right corner of the first page.

Problem G

(a) Show that the series

$$g(z)=\sum_{n=1}^{\infty}\left(\frac{1}{z-n\pi}+\frac{1}{z+n\pi}\right)$$

converges for every $z\neq m\pi$ for any nonzero integer m and that $g(0)=0$.
(b) The function

$$f(z)=\frac{1}{z}+\sum_{n=1}^{\infty}\left(\frac{1}{z-n\pi}+\frac{1}{z+n\pi}\right)$$

is thus well-defined for every $z\neq m\pi$ for any integer m. Show that

$$\lim_{z\longrightarrow 0} zf(z)=1, \qquad f(-z)=-f(z), \qquad f(z+\pi)=f(z), \qquad f(\pi/2)=0, \tag{1}$$

with the middle identities holding whenever either side is defined ($z\neq m\pi$ for any integer m).
Hint: use partial sums for the third equality; the other three are easy.
(c) What is the "simplest" function that satisfies all identities in (1)? (answer only)

Note: This all leads to $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$ as stated in 8.3; see solutions for more details.

Math Club Problem of the Month (November) (*not part of PS9*)

Let M be a square matrix consisting of 0's and 1's only. If there are precisely k 1's in each column, show that the determinant of M is divisible by k. Can the assumptions on the matrix be relaxed?