

MAT 127: Calculus C, Fall 2009 Homework Assignment 6

Problem Set 6 is due before the beginning of lecture on
Wednesday, 10/21 if enrolled in L01, L02
Thursday, 10/22 if enrolled in L03, L04

Please read Sections 7.6 and 8.1 thoroughly before starting on the corresponding problems below.

Problem Set 6: 7.6 2,4,6,9; 8.1 6,16,18,27; Problems E,F (see below)

Briefly explain your answer on all textbook problems and show your work on the letter problems

Please write your solutions legibly; the graders may disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name and lecture number in the upper-right corner of the first page.

Problem E

According to the book, the solutions $(x, y) = (x(t), y(t))$ to the system of differential equations

$$\begin{cases} \frac{dx}{dt} = ax - bxy \\ \frac{dy}{dt} = -cy + dxy \end{cases} \quad (1)$$

with certain constants $a, b, c, d < 0$ trace simple closed curves (loops) in the xy -plane. Let's see why.

(a) Divide the second equation in (1) by the first and solve the resulting equation obtaining $y = y(x)$ implicitly; in doing so assume that $x, y > 0$ (so only the first quadrant is considered).

(b) Fix the constant C in your general solution (this gives a specific solution of the equation in (a)). Show that the values of $x, y > 0$ that satisfy the equation lie in the interval $[m_C, M_C]$ for some $m_C, M_C > 0$. Furthermore, for each fixed $x > 0$ at most two values of $y > 0$ satisfy the equation; for each fixed $y > 0$ at most two values of $x > 0$ satisfy the equation.

(c) Assuming $x, y > 0$, show that $(x'(t), y'(t)) = 0$ if and only if $(x(t), y(t)) = (c/d, a/b)$.

For each value of the constant C , your solution in (a) is an equation for the curve traced by a solution $(x, y) = (x(t), y(t))$ to the system (1) in the xy -plane. By (b), this curve is contained in a square box, and every vertical and horizontal line intersects the curve at most twice. Thus, either the curve is closed and $(x(t), y(t))$ keeps on moving around it as t increases or $(x'(t), y'(t))$ approaches $(0,0)$ as $t \rightarrow \infty$ or $(x'(t_0), y'(t_0)) = (0,0)$ some t_0 (so that the path $(x(t), y(t))$ can reverse direction on the curve). By (c), the last two things can (and do) occur only on the "curve" containing the equilibrium point $(c/d, a/b)$; so each of the non-equilibrium solutions (1) keeps on going around some closed curve, containing the equilibrium point.

Problem F

Show (by induction) that the n -th Fibonacci number f_n of Example 3c in 8.1 is given by

$$f_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1 + \sqrt{5}}{2} \right)^n - \left(\frac{1 - \sqrt{5}}{2} \right)^n \right).$$

Is this consistent with the textbook's answer to 8.1 39b and why?