MAT 127: Calculus C, Fall 2009
Homework Assignment 5

Problem Set 5 is due before the beginning of lecture on
Wednesday, 10/07 if enrolled in L01
Monday, 10/05 if enrolled in L02
Tuesday, 10/06 if enrolled in L03, L04

Note the earlier due date (due to Midterm I; solutions will be available by noon on 10/07)

Please read *Notes on Second-Order Differential Equations* (called *Notes* below) thoroughly before starting on the problem set.

Problem Set 5: *Notes* 1,7,12,16; Problem D (below)
Show your work; correct answers without explanation will receive no credit, unless noted otherwise.

Please write your solutions legibly; the graders may disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name and lecture number in the upper-right corner of the first page.

Problem D

By part (c) of Problem B on HW2, the first-order differential equation

$$y' - by = f(x), \quad y = y(x), \qquad b = const,$$

can be solved by multiplying both sides by e^{-bx}. This equation then becomes

$$\big(\mathrm{e}^{-bx}y\big)' = \mathrm{e}^{-bx}f(x)$$

and can be solved by integrating both sides. Note that b is the root of the associated linear equation $r-b=0$. This approach has an analogue for second-order *in*homogeneous linear equations

$$y'' + by' + cy = f(x), \quad y = y(x), \qquad b, c = const. \tag{1}$$

(a) If r_1, r_2 are the two roots of the quadratic equation $r^2+br+c=0$ associated to (1), show that

$$\big(\mathrm{e}^{(r_1-r_2)x}(\mathrm{e}^{-r_1x}y)'\big)' = \mathrm{e}^{-r_2x}(y''+by'+cy). \tag{2}$$

By (2), equation (1) is equivalent to

$$\big(\mathrm{e}^{(r_1-r_2)x}(\mathrm{e}^{-r_1x}y)'\big)' = \mathrm{e}^{-r_2x}f(x), \quad y = y(x), \tag{3}$$

which can be solved by integrating twice.

(b,c) Find the general solutions $y=y(x)$ of the differential equations

$$\text{(b) } y''+5y'+4y = \mathrm{e}^{-x}, \qquad \text{(c) } y''+4y = 4\cos 2x.$$

Hint: In both cases, choose the order of the two roots wisely to get the simpler of the two possible versions of RHS in (3). In (c), it is simpler to replace $\cos 2x$ by $\mathrm{e}^{2\mathrm{i}x}$ and then take the real part of the general solution, which will be the general (real) solution to (c) because $\cos 2x$ is the real part of $\mathrm{e}^{2\mathrm{i}x}$ and all coefficients in the equation are real.