MAT 127: Calculus C, Fall 2009
Homework Assignment 2

Problem Set 2 is due by the beginning of lecture on
Wednesday, 09/16 if enrolled in L01, L02
Thursday, 09/17 if enrolled in L03, L04

Please read Section 7.2 thoroughly before starting on the problem set.

Problem Set 1: 7.2 3-6*,12,18,21,24; Problem B (below)
*Problems 3-6 should be written up as a single problem.
Show your work; correct answers without explanation will receive no credit, unless noted otherwise.

Please write your solutions legibly; the graders may disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name and lecture number in the upper-right corner of the first page.

Problem B

The Fundamental Theorem of Calculus from *Calculus B* provides a quick way of finding the general solution to a differential equations of the form

$$y' = f(x), \qquad y = y(x). \tag{1}$$

It turns out that every equation of the form

$$y' + a(x)y = f(x), \qquad y = y(x), \tag{2}$$

can be reduced to (1). Simply multiply both sides of (2) by a function $h=h(x)$ such that $h' = ah$:

$$h(x)y' + a(x)h(x)y = h(x)f(x) \quad \Longleftrightarrow \quad hy' + h'y = hf \quad \Longleftrightarrow \quad (hy)' = hf.$$

We can integrate both sides of the last equation and then divide by h. For example, multiplying

$$y' + 2xy = 2x, \qquad y = y(x), \tag{3}$$

by $h(x) = e^{x^2}$ gives

$$\mathrm{e}^{x^2}y' + 2x\mathrm{e}^{x^2}y = 2x\mathrm{e}^{x^2} \quad \Longleftrightarrow \quad \left(\mathrm{e}^{x^2}y\right)' = 2x\,\mathrm{e}^{x^2} \quad \Longleftrightarrow \quad \mathrm{e}^{x^2}y = \int 2x\,\mathrm{e}^{x^2}\,\mathrm{d}x = \mathrm{e}^{x^2} + C.$$

So the general solution of the differential equation (3) is $y(x) = 1 + Ce^{-x^2}$.

(a) Show that for any function $a = a(x)$, there exists a function $h = h(x)$ such that $h' = ah$.
Hint: see HW1 Problem A.
(b) Find the general solution of the differential equation

$$xy' + y = 2x, \qquad y = y(x).$$

What is the relation with the solution in Section 7.1, Exercise #1?
(c) Find the solution to the initial value problem

$$y' = x + y, \qquad y = y(x), \qquad y(0) = 1.$$

What is the relation with the numbers in the last column of the table at the bottom of p509 in the book?