MAT 127: Calculus C, Fall 2009
Homework Assignment 10

Problem Set 10 is due before the beginning of lecture on
Wednesday, 12/02 if enrolled in L01, L02
Thursday, 12/03 if enrolled in L03, L04

This homework assignment covers 1.5 weeks and so is longer than usual.

Please read Sections 8.6-8.7 thoroughly before starting on the problem set.

Problem Set 10: 8.6 6,14,29,30,35,36,38; 8.7 10*,20*,36*,42,45; Problems H,I (see below)
*also determine the radius and interval convergence

Please write your solutions legibly; the graders may disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name and lecture number in the upper-right corner of the first page.

Problem H

Use Taylor series to obtain *Euler's formula*:

$$\mathrm{e}^{\mathrm{i}t} = \cos t + \mathrm{i}\sin t.$$

This formula is used in solving second-order linear homogeneous differential equations with constant coefficients.

Problem I

(a) Let $p(x)$ be any polynomial in x and $n > 0$ any positive integer. Show that

$$\lim_{x \longrightarrow 0} x^{-n} p(x) \mathrm{e}^{-1/x^2} = 0.$$

(b) Show that the function $f = f(x)$ given by

$$f(x) = \begin{cases} \mathrm{e}^{-1/x^2}, & \text{if } x \neq 0, \\ 0, & \text{if } x = 0; \end{cases}$$

is smooth and its k-th derivative is a function of the form

$$f^{\langle k \rangle}(x) = \begin{cases} x^{-n_k} p_k(x) \mathrm{e}^{-1/x^2}, & \text{if } x \neq 0, \\ 0, & \text{if } x = 0, \end{cases}$$

where n_k is some positive integer and $p_k(x)$ is some polynomial in x. Conclude that the smooth function $f(x)$ does not admit a Taylor series expansion on any neighborhood of 0 (the Taylor series of f at $x=0$ does not converge to $f(x)$ for any $x\neq0$).