MAT 127: Calculus C, Fall 2009
Final Exam Information

Monday, 12/14, 8:15-10:45am

L01,L02: Old Chemistry Bldg 116	**L03,L04**: Earth&Space (ESS) Bldg 001

General Information

(0) **The exam will begin at 8:15AM** on the very first day of the exam week; do not miss it!!!!!

(1) It is **essential** that you show up to the location for the section you are registered in; please **note the swap in the locations** from the midterms. If you show up to the wrong location, your exam may not be accepted. **You must bring your Stony Brook ID card to the final exam.**

(2) Please show up no later than 8:10am. The exam will begin at 8:15am and you will not receive extra time if you show up after 8:10am.

(3) Please take every other seat starting with the front row. Once a row fills up, please take a seat *directly* behind another person (not diagonally from another person). You can put your bag and/or jacket on one of the seats next to you in the same row.

(4) Blank paper will be provided, in addition to an exam booklet (6 sheets stapled together).[1] The exam booklet should have sufficient space for solutions, but you can **staple** additional pages to it as needed. If you do so, please write your name and ID number on each additional sheet and indicate in the exam booklet where to find your solution. Any scrap paper that you not want to be graded should not be handed in (except separately from the exams, for recycling).

(5) No notes, books, calculators, or cell phones may be used during the exam. Please bring pencils/pens and an eraser. The *only* items that may be on your desk between 8:10am and 10:45am are pencils/pens, an eraser, your exam booklet, and the scrap paper provided by the proctors.

(6) When you receive the exam, please do not open it until the proctors say it is time to start. However, please do fill in your name and Stony Brook ID number and circle your section number on the front cover of the exam. The exact front cover of the exam is at the end of this handout.

(7) All problems on the exam should be stated unambiguously. If you feel there is an issue with a statement of a particular problem, please let a proctor know; however, the proctor will not confirm whether your interpretation of the problem is correct.

(8) Out of fairness to others, please do not open your exam booklet ahead of time and stop working when the time is over. Your exam score will be reduced by 5 points per minute of either violation.

[1] If the exams are photo-copied single-sided, instead of double-sided as requested, blank paper will still be available, but will be provided to you only upon request. You can write on all blank pages, but please indicate on the page containing the statement of a problem if your solution continues elsewhere. If you do not want something graded, please erase or cross it out, even if it is on a previously blank page.

(9) When you are finished with the exam or when the time is called (whichever comes first), please **hand your exam and your Stony Brook ID card** to the proctor, sign the photo roster under your picture, and get your ID card back.

(10) You can leave before the time is over, but please do so as quietly as possible and close the door very gently.

Before Final Exam

The grades for the second midterm and PS6-10 must be sorted out by Friday, 12/11; the grades for the first midterm and PS1-5 are no longer subject to change (as stated in *Midterm II Information*). The graders will have more detailed instructions than usual for grading PS11, which will hopefully reduce potential issues that may effect your letter grade for the semester.

The final exam will cover essentially all of the course: Sections 7.1-7.5, *Notes on Second-Order Differential Equations*, and Sections 8.1-8.7; Euler's method and the subject of Section 7.6 will not appear on the exam, and you do not need to remember any of the formulas from Sections 8.8, 8.9. You should re-read the relevant sections thoroughly, review *Course Summary I-III*, and study the solutions to PS1-11 (even if you did all/most problems correctly). Make sure you can do all problem set exercises (except possibly for those from Section 7.6) and some other related problems from the textbook, especially from *Concept Check*, *True-False Quiz*, and *Review Exercises* at the end of Chapters 7 and 8. *It is far better to be able to do the exercises correctly than to memorize the two chapters of the book or the three course summaries.*

Solutions to PS11 and 6 mini-quizzes will be posted late on Thursday, 12/10. Each of the quizzes will be *very* similar to a problem on the final (but may be slighter longer); the remaining two problems on the exam will be *very* similar to some of the exercises listed for the corresponding topic in *Types of Problems to Expect* below.

The final exams from Fall 05 and Spring 06 are available on the course website, along with solutions. This year's final exam will have some problems that are similar to some problems on these exams, but there will be differences as well. In particular, the Fall 05 and Spring 06 exams contain a problem requiring use of power series to solve a differential equation, which was the hardest problem on both exams. This problem concerned Section 8.10 in the second edition of the textbook; such a problem of course will not appear on your exam. This will be off-set by a higher dose of other power series problems, such as computation of sums of convergent series via Taylor series and approximating infinite sums by finite sums; an approximation problem, involving 8.9, is part of the Spring 06 exam. Thus, the two past final exams should not be viewed as "practice" or "sample" exams; instead, some problems on these exams are practice problems for some of the problems that will appear on your exam (see *Types of Problems to Expect* below).

If you have any questions, please come to office hours, MLC, a Residential Tutoring Center, and one or more of the following *additional* help sessions:

- **Office Hours**, Aleksey, Thursday, 12/3 and 12/10, 4-5pm, Math 3-111.
- **Review Session**, Caner, Friday, 12/11, 4-6pm, Math P-131. This will be a question-and-answer review session. If you have no questions, there will be no answers and no review

session.

- **Office Hours**, Qian, Saturday, 12/12, 5-7pm, Math 5-127.
- **Discussion Session**, Aleksey, Sunday, 12/13, 3-6pm, Math P-131. This means *you* discuss; this will not be a review session or a recitation. If there is nothing *you* want to discuss, there will be no discussion session. Depending on attendance and interest, the blackboard may be split into two for separate simultaneous discussions for the two parts of the course. Feel free to arrive and leave whenever you like; if you arrive after 3pm and have a question that has already been discussed, you may have to wait on it though.

You are welcome to come to any of the above, whether it is run by your instructor or not. At the same time, you should come only if you are expecting this to be helpful for you, so that you are not overly tired during the final exam for no good reason.

Note: If you find all of the front doors to the Math Tower locked on Saturday, 12/12, try all of the back doors. If you find all of the front doors locked on Sunday, 12/13, just knock on the windows of P-131.

After Final Exam

Detailed solutions to the final exam will be available on the course website on Monday (12/14) evening. Letter grade breakdowns for the exam may also be posted by then. Your score for the final exam should appear on blackboard by Monday evening as well; this score will be out of 150. The letter grades for the semester are unlikely to be submitted to solar before Tuesday evening, though the breakdowns may appear on the course website earlier.

While you can review your final exam, it is **not to be removed** from your instructor's office. As usual, you can stop by your instructor's office at the beginning of the Spring term to see your exam; you must bring a print-out of the solutions to the exam with you in order to do so. At that time, you'll be able to pick up your graded PS11, as well as any other problem sets and midterms you have not yet picked up; the grades for the midterms and PS1-10 will no longer be subject to change. If you'd like to discuss how your PS11 was graded with your instructor, you must have solutions to PS11 with you; any change in your PS11 grade is unlikely to be significant enough to effect the semester grade however.

If you are enrolled in L01/L03/L04, you can also stop by your instructor's office on Tuesday, December 15:

- Caner (L01): 1-3pm;
- Aleksey (L03/L04): 12:30-3:30pm.

If you would like to see your final exam (or pick up PS11) before the end of January, but absolutely can't come during your instructor's OHs on Tuesday, 12/15, please e-mail your instructor:

- a clear explanation why you can't make these OHs (e.g. *exam in BIO ... from ... to ...*);
- when on Monday evening and Tuesday you are able to come. While you are welcome to express a preference for some times over others, you may have to wait until the end of January to see your exam if you are not sufficiently flexible.

You should e-mail this information by Saturday, 12/12.

If you have done well in MAT 127, you should be ready for such courses as MAT 203/205 and 303/305. Some aspects of MAT 203/205 and 211 are useful for some aspects of 303/305, but these courses are not officially required for taking 303/305. Content-wise, MAT 203/205 is more of a continuation of 125 and 126, not 127; however, 127 is required for taking 203/205 as a transition from the lower level of 125 and 126. MAT 211 is independent of 127, and you may not find it any harder. If you'd like to discuss any of these courses with your instructor, the office hours on Tuesday, 12/15, would be a good time to do this.

Types of Problems to Expect

The final exam will have eight problems, but one of them will have two versions for you to choose from. The first problem will be worth 10 points; the remaining 7 problems will be 20 points each. All problems will be sub-divided into parts of specified weight. Four standard Taylor series will be provided at the top of the inside front cover of the exam (see last page of this handout), and you can use them (and their intervals of convergence) as appropriate. You must justify any other power series expansion you use, even if it it has appeared in class, in the textbook, or in the homework. For example, you might have seen that

$$\frac{x}{(1-x)^2} = \sum_{n=1}^{\infty} n x^n \,.$$

However, if you use this formula on the exam, in order to receive full credit you must justify it. This may mean deriving it from one of the standard formulas provided or directly from the function by computing its Taylor coefficients; quoting the book is not a justification in this case.

The list below should fairly accurately describe the problems that will appear on the exam; Euler's method problem, the content of 7.6, and your knowledge of the new formulas in 8.8 and 8.9 will not be tested on the final. Items (1)-(8) are listed roughly in the order they have appeared in the course, which is not the order in which they will appear on the exam. In particular, the Part I problems (items (1)-(3) below) will appear after the Part II problems because one of the former will have two versions and will be the last problem for this reason. The problems on your exam will be similar in style to the problems in the textbook, not to the letter problems on the problem sets; however, understanding solutions to the letter problems (especially J) might be helpful (understanding solutions to the textbook problem is absolutely essential). The hardest topics are perhaps (5) and (8); they are discussed in *Course Summary III*.

(1) Fundamental concepts in first-order differential equations. You will be given 4 first-order differential equations, 2 diagrams of direction fields, and 2 diagrams containing three graphs each. Each of the direction fields will correspond to precisely one of the 4 differential equations. All three graphs in a single diagram will be solution curves for precisely one of the differential equations, but it is possible that one of the curves will be a solution curve for more than one of the differential equations (**make sure** you understand what this sentence means now and ask before the exam if you are not 100% certain; the proctors will not clarify exam questions during the exam). There will a one-to-one match between the differential equations and the diagrams; you will be asked to determine what it is and explain your reasoning. This problem

can be done by elimination; each of the diagrams will have a feature which is incompatible with three of the equations (possibly different features for different equations), and so must correspond to the remaining equation. You need to be able to recognize constant solutions of a differential equation, understand special properties of autonomous equations, and see how a differential equation translates into the increasing/decreasing property of a solution and/or properties of the direction field. For example, the differential equation $y' = (x-2)(y-3)$ has precisely one constant solution, $y(x) = 3$ for all x, and thus cannot correspond to a diagram containing two horizontal lines. It also cannot correspond to a diagram containing a curve which is not strictly increasing in the region $x > 2$, $y > 3$, or to a direction field which is independent of x (such as Figure 10 on p507). Related examples: 7.1 10-12; 7.2 1-10,18; 7.CC 2; 7.TF 1; 7.RE 1; MIs05 2; MIf09 3,5.

(2) Separable equations and their applications. This problem may be of one of the following types:

- curve of specified slopes through specified point (including a sketch);
- orthogonal trajectories (including a sketch);
- mixing problem (with constant volume);
- exponential growth/decay equation;
- logistic growth equation.

Because 20 points is a lot to lose for not being able to set up a differential equation modeling a specific problem, this problem will have at least two parts. One part will ask you to show that the answer to the question is the solution to a specific, given, separable differential equation or initial-value problem (so you'll need to explain how you get from the question to the stated equation). Another part will involve solving the equation and possibly using the solution. So you may be able to get roughly half of the credit for this problem even if you cannot do the first part of it. Thus, you do not need to memorize the solutions to the exponential growth/decay equation and the logistic growth equation. This problem will have two versions, 8A and 8B (on different pages), which will involve two of the above five kinds of problems; your score for Problem 8 will be the higher of your scores on 8A and 8B. While there will be no grading penalty for attempting both versions of the problem, there will be no bonus for this either; so your time might be better spent on getting one of the two versions and the rest of the exam done right, instead of working on both versions. If you finish early, you should double-check that your solved the differential equation or the initial-value problem correctly by plugging your function into the differential equation and also checking that the initial condition is satisfied (if there is one). Make sure your final answer includes the correct physical units if appropriate (this may be appropriate in the last three types of problems, but not in the first two). All five types of problems are discussed in *Course Summary I*, but it is more important to be able to do the exercises correctly. Examples: 7.3 15,16,23-26 (sketch without graphing device), 35-38; 7.4 1-20; 7.5 1-8; 7.CC 6,7; 7.TF 5; 7.RE 9-16; MIf05 3,4; MIs06 1; MIf09 2; MIIs06 5; FEf05 2; FEs06 8. At the very least, you should be able to do the 10 problems from 7.3 (which cover 3 of the types of applications) and a few randomly chosen problems from each of 7.4 and 7.5.

(3) Second-order differential equations. You will be given one or more second-order linear homogeneous differential equations with constant coefficients or initial-value problems involving such equations. In each case, you will be asked to find either the general solution or a solution with specified initial conditions. Your answer should be in the real form (if the roots are

complex). If you are asked to find the general solution, *all* you need to do is to write down the corresponding quadratic equation (be careful if some coefficients are 0 though!), find its roots, and use the roots to write down the real form of the general solution depending on whether the two roots are real and distinct, real and the same, or complex conjugates of each other (see *Course Summary I*); your final answer will involve two constants, C_1 and C_2, in either of the three cases. If you are asked to solve an initial-value problem, you will also need to determine the two constants C_1 and C_2 using the initial values of y and y'; this involves solving a system of two linear equations in C_1 and C_2. Please test your readiness for the general-solution version of this problem by sitting down completely by yourself and writing down the general solutions to the 16 Notes problems (ignore initial conditions if there are any); this should take you 30-40 minutes, after which you should compare your answers with someone else in the class (and/or check them by plugging your solution back into the equation). Please test your readiness for the initial-value version of this problem by sitting down completely by yourself and writing down the solutions to Notes Problems 11-16; this should take you 60-80 minutes, after which you should compare your answers with someone else in the class (and/or check them by plugging your solution back into the equation and the initial conditions). Examples: Notes 1-16, MIf05 5; MIs06 3; MIf09 1; FEs06 7.

(4) Convergence and divergence of sequences and series. You will be given 5 sequences and/or series in this problem and asked to determine whether each of them converges or not, answering by clearly circling YES or NO (but not both). Each correct *answer* will be worth 2 points; no explanation will be expected or considered for grading purposes. If you cannot quickly determine the answer to a question in this problem, leave it until the end, since each question is worth only 2 points of 150; however, do not forget to come back to it and circle one of the possible answers, since there is no penalty for wrong guesses on this problem. This will be Problem 1 on the exam. Please test your readiness for the sequences parts of this problem by sitting down completely by yourself and writing yes/no (for converge/diverge) for each sequence in 8.1 9-34 (keeping in mind why you think so); this should take you 15-20 minutes, after which you should compare your answers with someone else in the class. Please test your readiness for the series parts of this problem by sitting down completely by yourself and writing yes/no (for converge/diverge) for each series in 8.2 3-6,13-30, 8.3 6-26, 8.4 5-8, 19-33 (keeping in mind why you think so); this should take you 60-90 minutes, after which you should compare your answers with someone else in the class. This problem is meant to be conceptual, testing whether you can tell convergence by the appearance of a sequence and series, not whether you can properly word a formal justification; see *Course Summary II,III*. Examples: 8.1 3-34,37,38a; 8.2 1-30; 8.3 3-27,38-40; 8.4 1-8,11,19-36; 8.CC 1-6; 8.TF 1-3,7-9,11,12,14-18; 8.RE 1-18,28,29; MIIf05 3,4c,5; MIIs06 2,4; MIIf09 1,3; FEf05 3,4abd; FEs06 3 (just determine convergence/divergence in all problems; no graphing devices).

(5) Estimating sums of series. In this problem you will be given more than one convergent series. You will be asked to explain why each of them actually converges and to estimate its sum with specified precision by using the minimal possible number of terms. The latter is done by choosing integer m such that the sum of the series starting with the $(m+1)$-th term is no larger than the specified precision and then summing the first m terms of the series to get the estimate. You'll need to justify why the integer m you choose provides an estimate for the series with the required precision. Based on what you have learned in class, you should know how to choose the required integer m (which is the substance of such problems) only

for two types of series $\sum_{n=1}^{\infty} a_n$:

- $a_n = f(n)$ for positive, continuous, decreasing to 0 function $f = f(x)$ defined for $x \geq 1$;
- $a_n \longrightarrow 0$, $|a_n| > |a_{n+1}|$, and the signs of terms a_n strictly alternate between + and −.

In both cases, the estimates are related to certain convergence tests for series. Make sure to check that the assumptions required for the two tests are satisfied (state the relevant properties in your setting and justify them if necessary); each of the two tests has three assumptions. You can expect one series of each kind. The correct value m will be sufficiently small so that you can add up the first m terms and leave your answer as a simple fraction (p/q for some integers p and q with no common factor). You should also be able to tell whether your estimate is an under- or over-estimate for the sum. Examples 8.3 28-32; 8.4 9,10,12-18; 8.6 25-29; 8.7 31,32,37,40; 8.8 10b; 8.9 23,24; 8.CC 7ac; 8.RE 25,26; FEf05 6b. At the very least, you should be able to do all problems from 8.3 and 8.4; they cover both cases.

(6) Taylor series, radius and interval of convergence of power series. You will be given more than one function and asked to find its Taylor series around the specified center. You will also need to determine the radius and interval of convergence for each of the resulting power series. Taylor series for four standard functions will be provided on the back of the front cover of the exam; one or more of them may be useful in some of the cases. Examples: 8.6 1-10; 8.7 5-16,19-26,28,29; 8.RE 24,30-41; FEf05 6a; FEs06 2.

(7) Power series, limits, and integration. Given a power series, you will be asked to determine its radius and interval of convergence, find a limit involving the function defined by this power series, and find an anti-derivative of a related function as a power series, along with its radius and interval of convergence. This problem will have 3 independent parts representing some aspects of 8.5, 8.6, and 8.7. Examples: 8.5 1-20; 8.6 12,13,21; 8.7 33,34,36-38,41-43; 8.CC 9; 8.RE 45.49; FEf05 5,7; FEs06 5.

(8) Convergence of series and computation of their sums via Taylor series. You will be given more than one convergent series and asked to explain why each of them converges and to find its sum. These two things can be done independently of each other. In order to compute their sums, you will need to represent each series as the Taylor series of some function $f = f(x)$ evaluated at some specific value of x (if you find a different way of obtaining the sum, that is fine). The provided standard series might again be useful; you may need to differentiate, integrate or multiply them by a power of x though. You can justify the convergence of each power series either by showing that the evaluation point lies in the interval of convergence of the power series or by using one or more of the many convergence tests in 8.3 and 8.4; in the latter case the Ratio Test is likely to be your best bet, as it usually works best with power series. As was the case on Midterm II, you do not have to memorize the names of the tests, but you have to make it clear what test you are using and check the required assumptions (see *Midterm II Info* for examples). Examples: 8.6 11,36; 8.7 50-54; FEf05 6.

Above *.CC, *.TF, *.RE refer to the *Concept Check*, *True-False Quiz*, and Review *Exercises* at the end of Chapter *; Notes refers to *Notes on Second-Order Linear Differential Equations*; MIf05, MIIf05, FEf05 and MIs06, MIIs06, and FEs06 refer to the three exams from Fall 05 and Spring 06 (available from the course website).

MAT 127 Final Exam

December 14, 2009
8:15-10:45am

Name:____________________________ **ID:**__________________

Section:	L01	L02	L03	L04	(circle yours)
	MWF 9:35-10:30am	MW 5:20-6:45pm	TuTh 2:20-3:40pm	TuTh 5:20-6:40pm	

DOO NOT OPEN THIS EXAM YET

Instructions

(1) This exam is closed-book and closed-notes; no calculators, no phones.
(2) Please write legibly. Circle or box your final answers.
(3) Show your work. Correct answers only will receive only partial credit.
(4) Simplify your answers as much as possible.
(5) Leave your answers in exact form (e.g. $\sqrt{2}$, not ≈ 1.4).
(6) If you need more blank paper, ask a proctor.
(7) Please write your name and ID number on any additional sheets you'd like to be graded and staple them to the back of the exam (stapler provided); indicate in the exam that the solution continues on the attached sheets.
(8) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.

Out of fairness to others, please **stop working and close the exam as soon as the time is called**. A significant number of points will be taken off your exam score if you continue working after the time is called. You will be given a two-minute warning before the end.

Some Taylor Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{if} \ \ |x| < 1 \qquad\qquad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad\qquad \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

to receive full credit, justify any other power series expansion you use

1 (10pts)	
2 (20pts)	
3 (20pts)	
4 (20pts)	
Subtotal (70pts)	

5 (20pts)	
6 (20pts)	
7 (20pts)	
8A/B (20pts)	
Subtotal (80pts)	

Total (150pts)	