

MAT 127: Calculus C, Fall 2009

Midterm II Information

Wednesday, 11/04, 8:30-10:00pm

L01,L02: Earth&Space (ESS) Bldg 001

L03,L04: Old Chemistry Bldg 116

General Information

- (1) It is **essential** that you show up to the location for the section you are registered in. All locations have limited seating, the proctors will have a limited number of exams at each location, and if your exam gets mixed in with a different section, your exam grade may not be recorded. *You must bring your Stony Brook ID card to the exam.*
- (2) Please show up no later than 8:25pm. The exam will begin at 8:30pm and you will not receive extra time if you show up after 8:25pm.
- (3) Please take every other seat starting with the front row. Once a row fills up, please take a seat *directly* behind another person (not diagonally from another person). You can put your bag and/or jacket on one of the seats next to you in the same row.
- (4) Blank paper will be provided, in addition to an exam booklet (4 sheets stapled together). The exam booklet should have sufficient space for solutions, but you can staple additional pages to it as needed. If you do so, please write your name and ID number on each additional sheet and indicate in the exam booklet where to find your solution. Any scrap paper that you not want to be graded should not be handed in (except separately from the exams, for recycling).
- (5) No notes, books, calculators, or cell phones may be used during the exam. Please bring pencils/pens and an eraser. The *only* items that may be on your desk between 8:25pm and 10:00pm are pencils/pens, an eraser, your exam booklet, and the scrap paper provided by the proctors.
- (6) When you receive the exam, please do not open it until the proctors say it is time to start. However, please do fill in your name and Stony Brook ID number and circle your section number on the front cover of the exam. The exact front cover of the exam is at the end of this handout.
- (7) All problems on the exam should be stated unambiguously. If you feel there is an issue with a statement of a particular problem, please let a proctor know; however, the proctor will not confirm whether your interpretation of the problem is correct.
- (8) Out of fairness to others, please do not open your exam booklet ahead of time and stop working when the time is over. Your exam score will be reduced by 5 points per minute of either violation.
- (9) When you are finished with the exam or when the time is called (whichever comes first), please take it to the front along with your Stony Brook ID card. Put the exam in the pile for your section and sign the photo roster under your picture immediately after.

(10) You can leave before the time is over, but please do so as quietly as possible and close the door very gently.

Before Midterm II

Note that PS8 is due two days earlier than usual (except in L01). This will make it possible to post solutions to PS8 before noon on Wednesday, 11/04.

The midterm will cover Sections 7.6 and 8.1-8.3 from the textbook. You should re-read these sections thoroughly, review *Course Summary II*, and study the solutions to PS6-8 (even if you did all/most problems correctly). Make sure you can do all problem set exercises from the above four sections and some other related problems from the textbook.

The second midterms from Fall 05 and Spring 06 are available on the course website, along with solutions. Please try doing these midterms in 90 minutes (each) before looking at the solutions. The second midterm in this class will be similar in many aspects to these midterms, though there will be some differences. In particular, the Spring 06 midterm contains a plain logistic equation problem, which will not appear on your exam (see more details below); you should still be able to do this midterm though.

The grades in MAT 127 have had a history of dropping significantly from Midterm I to Midterm II. Section 7.6 is a hard topic being heavily graphics and graphics of rather difficult kind. Sections 8.1-8.3 often lead to confusion between sequences and series, the corresponding notions of convergence, and the corresponding convergence/divergence tests. This is rather avoidable while working on exercises from these sections, as sequences and series are kept separate there. It is thus essential to do exercises from the *Review* portion of Chapter 8 (*Concept Check*, *True-False Quiz*, and *Review Exercises*) as well as the old midterms under test conditions. Try to start studying for the exam as early as possible so that you can get some rest before the exam and not be exhausted while taking it (this appears to have had negative effect on quite a number of Midterm I scores).

If you received an F or D/C- on the first midterm (or a low C and your homework scores are low), you should do the Fall 05 second midterm under test conditions by Sunday, November 1. If you do poorly on it (and be honest with yourself in comparing your work with the solutions), you should probably withdraw from the class while you can. The *last* day to do so is Monday, November 2; you will receive a W on your transcript if you withdraw at this point, but presumably this is preferable to an F.

If you have any questions, please come to office hours (lots of them on Wednesdays!), MLC, and/or a Residential Tutoring Center. If you do not do well on the second midterm (which covers only 4 sections), it is likely to be very hard to compensate for this on the final (which will be cumulative).

Note that any possible issues concerning your grades on Midterm I and PS1-5 must be resolved before Midterm II. Midterm I and PS1-5 grades will not be changed after November 4 even if your score was simply tallied incorrectly.

After Midterm II

Detailed solutions to the midterm will be available on the course website on Thursday morning; please print these out before the following lecture. If your total exam score was incorrectly tallied, please let your instructor know.

Before raising questions about how your exam was graded, you **must** read the solutions to the exam. Each problem is intended to be graded according to a fixed grading scheme, which will be outlined in the solutions. Errors (deviations from the scheme) can occur in grading, and you are welcome to discuss your score on each specific problem on the exam with the primary grader for the problem (bring along your exam **and** solutions). He/she will take this opportunity to check that the *entire* problem on your exam (and not just the part you are concerned about) was graded according to the grading scheme. This may change your overall score for the problem, either raising or lowering it, likely marginally, if at all. If your overall exam grade is changed (up or down), it will then be updated on your exam and on *blackboard*. If you decide to discuss the grading of a specific problem with an instructor, the grading of any of the remaining problems on your exam may be subject to re-evaluation at the instructor's discretion.

Your grade will be changed under the following circumstances only:

- if it is contrary to the grading scheme outlined in the solutions;
- if this outline does not address the issue in question and your score is contrary to the spirit of the scheme or is inconsistent with how another exam was graded. In the latter case, you have to come with the student whose exam was graded differently; it may be the case that his/her score will need to be adjusted down, instead of yours up.

While this re-grading policy may appear overly harsh, its only aim is to increase the likelihood that your overall exam score accurately reflects your work on the exam; this is only fair to you and the rest of the class.

One of the aims in grading the exams is consistency. Another aim is for the number of points taken off for an error to be appropriate in relation to the significance of the error to the entire problem as well as to the content of the class. In particular, the same error may carry different penalties on different problems. For example, the 20-point Problem 2 on Midterm I was rather simple; answering the actual question being asked and using $(\ln 16)/(\ln 2) = 4$ thus constituted significant portions of the problem. In something like Problem 5 on Midterm I, their relative significance would have been less. At the same time, expressions like $\ln |2|$ and $(\ln 16)/(\ln 2)$ in final answers constitute fundamental errors, as the first indicates you have no understanding of what absolute value means while the second that you have no understanding of logs. Errors such as $\ln 35 - \ln 5 = \ln 30$ or $(\ln 35)/(\ln 5) = \ln 7$ are also very serious (in fact, $\ln 35 - \ln 5 = \ln 7$); this would be another matter in MAT 123 or even MAT 125. Stating that a (random) solution of an autonomous equation $y' = f(y)$, $y = y(x)$, is independent of x shows fundamental misunderstanding of the course, as solutions are functions of x (yes, some of them may be constant functions, but most are not if $f(y) \neq 0$). All such errors could carry somewhat heavier penalties than you might expect.

On the other hand, the penalty for plain computational errors is fairly light. For example, if you forget a minus sign going from one line to the next, divide 1000 by 8 and get 120, or add $7/8$ and

17/64 incorrectly, you are unlikely to lose more than 1 point. Carryover errors are generally not penalized. However, there are cases when a minor error fundamentally alters the problem, making it much easier; replacing $(y^2 - 1)^{-1}dy$ by $(y^2 - 1)dy$ in Problem 5 on Midterm I is an example. In such cases, you may lose most, if not all, of the points allocated to the problem; in the case of Problem 5 on Midterm I, you could have avoided this by checking that your solution actually solved the original equation.

Penalties that are heavier than one might expect are fairly rare. If you are reasonably expecting an A/A- in this course, you should be extremely unlikely to make such a mistake and thus should not be concerned about this. If you are just hoping for a C, try to do correctly and completely what you know you can do instead of writing something for every part of every problem, including those you have no clue about.

Background Material

You should be familiar with and know how to use the two FTCs, chain and product rules, integration-by-parts and change-of-variables formulas, and partial fractions. You may also encounter integrals like

$$\int xe^{rx} dx, \quad \int \frac{1}{x(x+1)} dx.$$

Types of Problems to Expect

The second midterm will have five problems, not necessarily of equal weight, with most problems sub-divided into parts of specified weight. Your midterm will be similar in many aspects to the second midterms in Fall 05 and Spring 06, but there will be some differences and your midterm will not have a plain logistics equation problem (see (1) below for more details). The ability to solve first and second-order equations is not required for the second midterm (but it will be tested on the final). You will not be asked to estimate the sum of a convergent series to specified precision on the second midterm (but may be asked on the final). The list below should fairly accurately describe the problems that will appear on the exam. Items (1)-(5) below are listed roughly in the order they have appeared in the course, which is not the order in which they will appear on the exam; in particular, the item (1) problem will appear last to make it simpler to use the back of the exam for it. If you are asked to sketch anything, label the coordinate axes correctly; missing and incorrect labels will result in identical penalties. The problems on your midterm will be similar in style to the problems in the textbook, not to the letter problems on the problem sets; however, understanding solutions to the letter problems might be helpful (understanding solutions to the textbook problems is necessary).

- (1) Systems of 2 autonomous first-order differential equations and modeling two-species interactions. Given such a system, you should be able to determine what type of interactions it is modeling and/or which species corresponds to each of the two variables. You should be able to find the equilibrium points or constant solutions of such a system and explain their significance. Given a phase trajectory, you need to be able to sketch graphs of the corresponding functions and vice versa; in both cases, the axes should have appropriate labels and you need to be

able to explain the sketching process. This may require you to estimate coordinates of some points. Your estimates should be reasonable and consistent. For example, if a coordinate of some point appears to be roughly half way between 200 and 300 and 250 is not marked, anything between 230 and 270 would be reasonable. However, if one point lies to the left of another, your estimate for the horizontal coordinate of the former should be smaller than for the latter. If you are given that one of the coordinates of a point is 250, you should not change it to 255. You may want to use color pencils and/or pens, but please do not use red or green pens; red or green pencils are ok if they are clearly distinguishable from pens. You do not need to remember the explicit solutions to the exponential growth/decay equation and the logistic equation. However, you must be able to recognize these equations and know what happens to their solutions as the independent variable $t \rightarrow \infty$; in particular, you need to know what the equilibrium points are. Examples: 7.6 1-10; 7.CC 6-8; 7.TF 5; 7.RE 20,21; MIIIf05 1; MIIIs06 1.

- (2) **Convergence and divergence of sequences.** You will be given more than one sequence in this problem. In each case, you will need to decide whether the sequence converges or not and to justify your answer. You may be able to do so in a fairly direct way or may need to use one or more of the boxed statements in 8.1. Examples: 8.1 1-34,37,38a; 8.CC 1,2; 8.TF 3,11,14-16,18; 8.RE 1-8; MIIIf05 3; MIIIf06 2 (just determine convergence/divergence; no graphing devices).
- (3) **Computation of limits of convergent sequences.** You will be given more than one convergent sequence in this problem and asked to find their limits, justifying your answer along the way. You will not need to explain why the sequence converges before computing its limit; this may or may not be of any benefit, depending on the sequence. You can reasonably expect something very standard as well as something less standard, in the style of 8.1 39b,40,46-48 (they all involve the same principle). Examples: 8.1 3,9-34,38c,39b,40,46-48; 8.RE 1-8; MIIIf05 3; MIIIs06 2 (compute limits for convergent sequences only; no graphing devices).
- (4) **Computation of limits of convergent series.** In this problem, you will be given one or more convergent series and asked to find its/their sum(s) as well as a question like 8.1 31-34 (they all involve the same principle). You will not need to explain why each of the given power series converges before computing its sum. You may need to use the series rules, sum of geometric series, and/or telescoping cancellation possibly along with partial fractions as in Example 6 in 8.2. Examples: 8.2 9-34,39,41,42,45; 8.CC 4; 8.TF 10; 8.RE 19-23; MIIIf05 4ab; MIIIs06 3 (find sums of convergent series only; no calculators).
- (5) **Convergence and divergence of series.** You will be given more than one series in this problem. In each case, you will need to decide whether the series converges or not and to justify your answer. You may be able to do so in a fairly direct way or may need to use one or more of the *many* boxed statements in 8.2 and 8.3. Examples: 8.2 1,2,11-30,35-39; 8.3 1,3-27,38-40; 8.CC 3,5-7; 8.TF 1,2,4-9,12,17; 8.RE 9-18; MIIIf05 4c,5; MIIIs06 4 (just determine convergence/divergence; no calculators).

Above *.CC, *.TF, *.RE refer to the *Concept Check*, *True-False Quiz*, and *Review Exercises* at the end of Chapter *; MIIIf05 and MIIIs06 refer to the second midterms from Fall 05 and Spring 06 (available from the course website).

While you need to know, understand, and be able to use the contents of all of the boxes on pp 559-582, you do not need to memorize the name associated with each box. However, your argument must it clear what you are using. For example, if you are asked whether the sequence $a_n = 1 + (-1)^n/n$ converges or diverges, you could say:

converges because $1 - 1/n \leq a_n \leq 1 + 1/n$ and the sequences $b_n = 1 - 1/n$ and $c_n = 1 + 1/n$ converge to the same limit, which is 1.

This would receive full credit; what has just been used is the *Squeeze Theorem for Sequences* on p560, but you do not have to state it. By the same theorem, $\lim_{n \rightarrow \infty} a_n = 1$.

In some cases, there are different ways to justify your answer. For example, suppose you are asked whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$ converges or diverges. You could say

converges because $0 \leq \frac{1}{n^2 + 1} \leq \frac{1}{n^2}$ and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

This uses the *Comparison Test* on p580 and the *p-Series Test* on p579; the former requires the terms in the series to be nonnegative, and so you should make it clear that you are aware of this. You could instead say

converges because $\frac{1}{n^2 + 1}, \frac{1}{n^2} > 0$, $\lim_{n \rightarrow \infty} \frac{n^2 + 1}{n^2} = \lim_{n \rightarrow \infty} (1 + 1/n^2) = 1$ and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

This uses the *Limit Comparison Test* on p582 and the *p-Series Test* on p579. A third possibility is

converges because $f(x) = \frac{1}{1 + x^2}$ is a continuous, positive, and decreasing function on $[1, \infty)$ and

$$\int_1^{\infty} \frac{1}{1 + x^2} dx = \arctan x \Big|_{x=1}^{\infty} = \lim_{x \rightarrow \infty} \arctan x - \arctan 1 = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

is finite.

This uses the *Integral Test* on p578. The last argument requires knowing the integral of $1/(1+x^2)$ and that $\arctan x$ approaches a finite value as $x \rightarrow \infty$. If properly worded, it does not actually require knowing that this finite value is $\pi/2$ or that that $\arctan 1 = \pi/4$. The latter is avoidable by stating that $\arctan x$ is defined for all x . Whenever your answer can be justified in different ways, you should give *only one* correct and complete justification. If you give more than justification and every one of them is correct and complete, you will receive full credit. However, if one of them contains an error, you will lose points even if another explanation is correct and complete.

MAT 127

Midterm II

November 4, 2009
8:30-10:00pm

Name: _____

ID: _____

Section: L01 L02 L03 L04 (circle yours)
 MWF 9:35-10:30am MW 5:20-6:45pm TuTh 2:20-3:40pm TuTh 5:20-6:40pm

DO NOT OPEN THIS EXAM YET

Instructions

- (1) This exam is closed-book and closed-notes; no calculators, no phones.
- (2) Please write legibly. Circle or box your final answers.
- (3) Show your work. Correct answers only will receive only partial credit.
- (4) Simplify your answers as much as possible.
- (5) Leave your answers in exact form (e.g. $\sqrt{2}$, not ≈ 1.4).
- (6) If you need more blank paper, ask a proctor.
- (7) Please write your name and ID number on any additional sheets you'd like to be graded and **staple** them to the back of the exam (stapler provided); indicate in the exam that the solution continues on the attached sheets.
- (8) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.

Out of fairness to others, please **stop working and close the exam as soon as the time is called**. A significant number of points will be taken off your exam score if you continue working after the time is called. You will be given a two-minute warning before the end.