MAT 531 SPRING 16 HOMEWORK 9

Due Tuesday, Apr 12

1. Let (e^1, e^2, e^3) be the standard dual basis for $(\mathbb{R}^3)^*$. Show that $e^1 \otimes e^2 \otimes e^3$ is not equal to a sum of an alternating tensor and a symmetric tensor.

2. Define a 2-form Ω on $\mathbb{R}^3$ by

$$\Omega = x dy \wedge dz + y dz \wedge dx + z dx \wedge dy.$$

(a). Compute Ω in spherical coordinates (ρ, φ, θ) where $(x, y, z) = (\rho \sin\varphi \cos\theta, \rho \sin\varphi \sin\theta, \rho \cos\varphi)$.

(b). Compute $d\Omega$ in both Cartesian and spherical coordinates and check that both expressions give the same 3-form.

(c) Compute the restriction $\Omega|_{S^2} = \iota^*\Omega$ using coordinates (φ, θ), (on their domain of validity). Here $\iota : S^2 \to \mathbb{R}^3$ is the standard inclusion of the unit sphere $S^2(1)$ into $\mathbb{R}^3$.

(d). Show that $\Omega|_{S^2}$ is nowhere zero.

3. Spivak, # 11, Chapter 7

4. Spivak, # 21, Chapter 7