MAT [illegible] HW [illegible]

1) Suppose $W(x,t) = U(x,t) - V(x)$

where $V(x) = \lim_{t\to\infty} U(x,t)$ [illegible]

So W must satisfy same [illegible]

[illegible]

2) [illegible]

$V(x) = A \ln [illegible]$

[illegible]

MAT 341 HW 3

2.2

(4) Suppose $W(x,t) = U(x,t) - V(x)$

Where $V(x) = \lim_{t\to\infty} U(x,t) \Rightarrow \frac{\partial^2 V}{\partial x^2} = \frac{1}{k}\frac{\partial V}{\partial t} = 0$

$\therefore$ So we get after some manipulations

$$\begin{cases} \frac{\partial^2 W}{\partial x^2} = \frac{1}{K}\frac{\partial W}{\partial t} \quad \text{—— (i)}, \\ W(0,t) = U(0,t) - V(0) = 0 \quad (t>0) \text{ — (ii)}, \\ -K\frac{\partial W}{\partial x}(a,t) = h\,W(a,t) \quad \text{—— (iii)} \\ \text{and} \end{cases}$$

$$W(x,0) = U(x,0) - V(x) = f(x) - T_0 - \frac{xh(T_1 - T_0)}{k + ha} \text{ — (iv)}$$

These are the eqns of transient temp. distr.

(5)

$$\lim_{t\to\infty} \frac{\partial}{\partial x}\left(K\left(\frac{\partial u}{\partial x}\right)\right) = 0 \qquad \left(\text{as } \lim_{t\to\infty}\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = 0\right)$$

$$\therefore \frac{\partial}{\partial x}\left(K\left(\frac{\partial V}{\partial x}\right)\right) = 0 \qquad \text{(By taking the limit inside)}$$

$$\Rightarrow K\frac{\partial V}{\partial x} = C_1 \Rightarrow \frac{\partial V}{\partial x} = \frac{C_1}{k_0 + \beta x}$$

$$\Rightarrow V(x) = A\ln(K_0 + \beta x) + B \quad (A, B \text{ cons})$$

We know $V(0) = A\ln K_0 + B = T_0$ — (1)

and $V(a) = A\ln(K_0 + a\beta) + B = T_1$ — (2)

Solving (1) and (2) yields

$$A = \frac{T_1 - T_0}{\ln\left(\frac{K_0 + a\beta}{K_0}\right)} \quad \text{and} \quad B = T_0 - \frac{T_1 - T_0}{\ln\left(\frac{K_0 + a\beta}{K_0}\right)}\ln K_0$$

[illegible]

$b_n = \frac{2}{L}\int_0^L$ [illegible] $\sin\left(\frac{n\pi x}{L}\right) dx$

[illegible]

(3) $u(x,0) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right) = $ [illegible]

$b_n = \frac{2}{L}\int_0^L$ [illegible] $\sin\left(\frac{n\pi x}{L}\right) dx$

$= \frac{2}{L}\Big[$ [illegible] $\Big]_0^L$

[illegible]

$u(x,t) = \sum b_n \sin\left(\frac{n\pi x}{L}\right)$ [illegible]

[illegible]

2.3

(5)

$$W(x,0) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{a}\right) = T_0$$

$$\therefore\ b_n = \frac{2}{a}\int_0^a T_0 \sin\left(\frac{n\pi x}{a}\right)dx$$ (By applying f.s. Formula)

$$= \frac{2}{a}T_0 \left(\frac{a}{n\pi}\left(-\cos\frac{n\pi x}{a}\right)\right|_0^a$$

$$= \frac{2}{n\pi}(1-\cos n\pi)\,T_0$$

$$\therefore\ W(x,t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{a}\right)\exp\left(-\left(\frac{n\pi}{a}\right)^2 kt\right)$$

(6)

$$W(x,0) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{a}\right) = \beta x$$

$$b_n = \frac{2}{a}\int_0^a \beta x \sin\left(\frac{n\pi x}{a}\right)dx$$

$$= \frac{2}{a}\left|\beta x\left(-\frac{a}{n\pi}\cos\frac{n\pi x}{a}\right)\right|_0^a + \frac{a}{n\pi}\frac{2\beta}{a}\int_0^a \cos\left(\frac{n\pi x}{a}\right)dx$$

vanishes

(By Integrating By parts)

$$= \frac{-2a\beta\cos(n\pi)}{n\pi}$$; $W(x,t)$ is as in (5)

(7)

Similar as above here

$$b_n = \frac{2}{a}\int_0^a \beta(a-x)\sin\left(\frac{n\pi x}{a}\right)dx$$

$$= \frac{2}{a}\left|\beta(a-x)\left(-\frac{a}{n\pi}\cos\frac{n\pi x}{a}\right)\right|_0^a + \frac{a}{n\pi}\frac{(-2\beta)}{a}\int_0^a \cos\left(\frac{n\pi x}{a}\right)dx$$

vanishes

$$= \frac{2a\beta}{n\pi}$$

$$b_n = \frac{2}{l}\int_0^l (\ldots) \sin\left(\frac{n\pi x}{l}\right) dx$$

[illegible]

$$= \frac{8 l^2}{n^3\pi^3} \sin(n\pi/2)$$

[illegible]

(8) Here $W(x,0)$ is piecewise smooth

$$W(x,0) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{a}\right) = \begin{cases} \frac{2T_0 x}{a} & 0<x<\frac{a}{2} \\ \frac{2T_0(a-x)}{a} & \frac{a}{2}<x<a \end{cases}$$

$$b_n = \frac{2}{a}\int_0^a W(x,0)\sin\left(\frac{n\pi x}{a}\right)dx$$

$$= \frac{2}{a}\left\{\int_0^{a/2}\frac{2T_0 x}{a}\sin\left(\frac{n\pi x}{a}\right)dx + \int_{a/2}^{a}\frac{2T_0(a-x)}{a}\sin\left(\frac{n\pi x}{a}\right)dx\right.$$

$$= \frac{2}{a}\left\{\left|\frac{2T_0 x}{a}\left(-\cos\left(\frac{n\pi x}{a}\right)\frac{a}{n\pi}\right)\right|_0^{a/2}\right.$$

$$+\left|\frac{2T_0}{a}\sin\left(\frac{n\pi x}{a}\right)\right|_0^{a/2}\left(\frac{a}{n\pi}\right)^2$$

$$+\left|\frac{2T_0(a-x)}{a}\left(-\cos\left(\frac{n\pi x}{a}\right)\frac{a}{n\pi}\right)\right|_{a/2}^{a}$$

$$+\left|-\frac{2T_0}{a}\sin\frac{n\pi x}{a}\right|_{a/2}^{a}\left(\frac{a}{n\pi}\right)^2$$

$$= \frac{2}{a}\left\{\cancel{\frac{T_0 a}{n\pi}\left(-\cos\frac{n\pi}{2}\right)} + \frac{2T_0}{a}\sin\left(\frac{n\pi}{2}\right)\left(\frac{a}{n\pi}\right)^2\right.$$

$$\left. - \cancel{\frac{T_0 a}{n\pi}\left(-\cos\frac{n\pi}{2}\right)} + \frac{2T_0}{a}\sin\left(\frac{n\pi}{2}\right)\left(\frac{a}{n\pi}\right)^2\right\}$$

$$= \frac{8T_0}{n^2\pi^2}\sin(n\pi/2)$$

$W(x,t)$ is as in other problems.