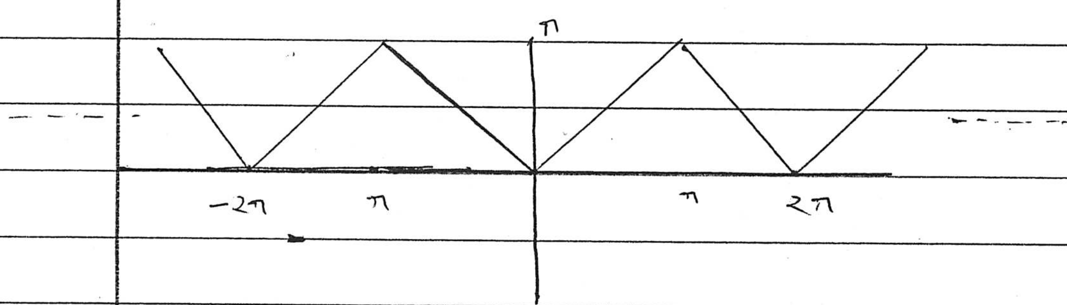
1.1

(1b)

$$f(x) = |x| \qquad -\pi < x < \pi$$



The coefficients are

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |x|\,dx = \pi/2$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$$

$$= \frac{2}{\pi}\int_{0}^{\pi} x\cos(nx)\,dx$$

(By Integration By Parts) $= \dfrac{2}{\pi}\left[\left.\dfrac{x\sin(nx)}{n}\right|_0^{\pi} - \displaystyle\int_0^{\pi}\dfrac{\sin(nx)}{n}\,dx\right]$

$$= \frac{2}{\pi n^2}\left[(-1)^n - 1\right]$$

$\therefore \quad a_n = 0 \qquad$ if n even

$\quad\;\; = -\dfrac{4}{\pi n^2} \qquad$ if n odd

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} |x| \sin(nx)\, dx \qquad = 0$$

(even) x (odd) = (odd) function

as its an integral of an odd fn.

$$\therefore\ |x| \sim \frac{\pi}{2} + \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \left(\frac{-4}{\pi n^2}\right) \cos(nx)$$

(7) (a) $f(x) = \cos^2 x$

$$= \frac{1}{2} + \frac{\cos(2x)}{2} \quad \leftarrow \text{The series}$$

(b) $f(x) = \sin(x - \pi/6)$

$$= \sin x \cos\left(\frac{\pi}{6}\right) - \cos x \sin(\pi/6)$$

$$= \frac{\sqrt{3}}{2} \sin x - \frac{1}{2}\cos x \quad \leftarrow$$

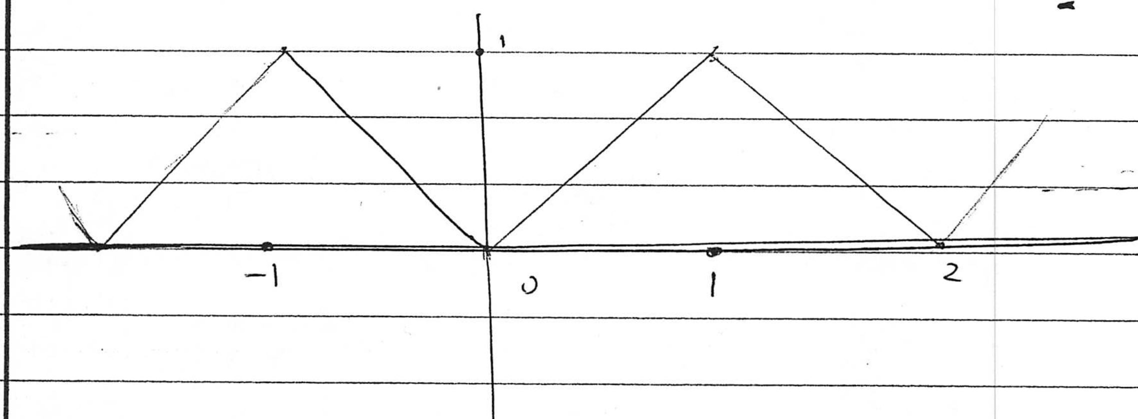
(c) $f(x) = \sin x \cos 2x$

$$= \frac{1}{2}\left[\sin(x + 2x) + \sin(x - 2x)\right]$$

$$= \frac{1}{2}\sin(3x) - \frac{1}{2}\sin x$$

1.2

(1a) $f(x) = |x| \qquad -1 < x < 1$



$$a_0 = \frac{1}{2a}\int_{-a}^{a} |x|\,dx = \frac{1}{2} \qquad a = 1$$

$$b_n = \frac{1}{a}\int_{-a}^{a} |x| \sin\left(\frac{n\pi x}{a}\right) dx = 0 \qquad a = 1$$

$$a_n = \frac{1}{a}\int_{-a}^{a} |x| \cos\left(\frac{n\pi x}{a}\right) dx$$

$$= \int_{-1}^{1} |x| \cos(n\pi x)$$

$$= \frac{2}{n^2\pi^2}\left[(-1)^n - 1\right] \quad \text{by earliar method}$$

$\therefore \; a_n = 0$ if n even

$$= \frac{-4}{n^2\pi^2}$$

$$\therefore \; |x| \sim \frac{1}{2} + \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \left(\frac{-4}{n^2\pi^2}\right) \cos(n\pi x)$$

(10d)

(4)

By Defn.

$$\cosh(x) = \frac{1}{2}(e^x + e^{-x}) \quad (= \text{even})$$

$$\sinh(x) = \frac{1}{2}(e^x - e^{-x}) \quad (= \text{odd})$$

So $e^x = \cosh(x) + \sinh(x)$

gives the desired decomposition

(8) Say we have a periodic continous function then Yes. (If we assume the fn. has a everywhere convergent Fourier series)

In the case all the sine coefficients of a function vanish

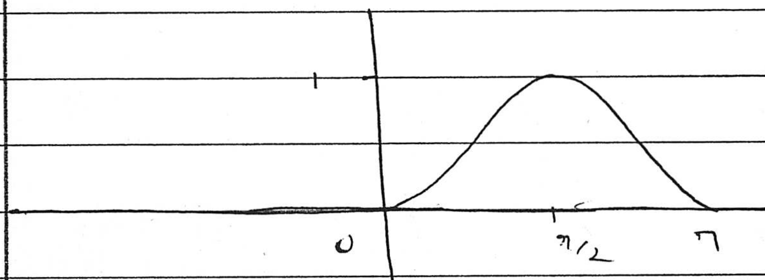
$$\therefore f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{a}\right) \qquad -a < x < a$$

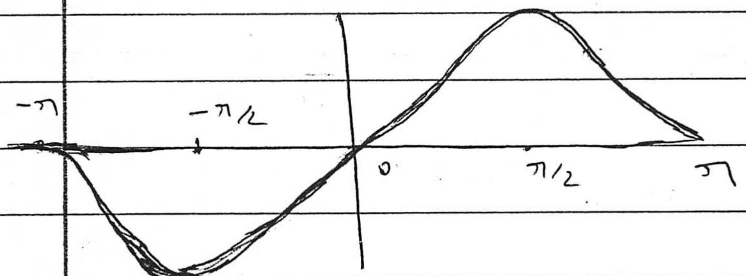
Then $\therefore$ $f(x) = f(-x)$

But it may not happen that a function has a everywhere convergent fourier series. In that case this is not true. For example — Let us take any const fn. with finitely many discontinuity

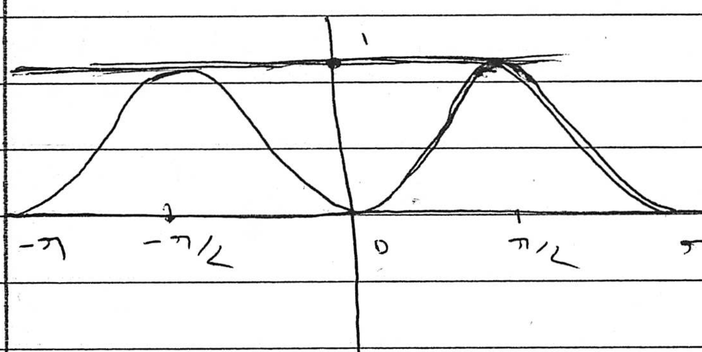
(10d)

$$f(x) = \sin(x) \qquad 0 < x < \pi$$





odd extension



even extension

(11d)

$$f(x) = \sin(x) \qquad 0 < x < \pi$$

The cosine series is

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{a}\right) \qquad 0 < x < a$$

$$a_0 = \frac{\int_0^{\pi} \sin x \, dx}{\pi} = \frac{2}{\pi}$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} \sin x \cos(nx) \, dx$$

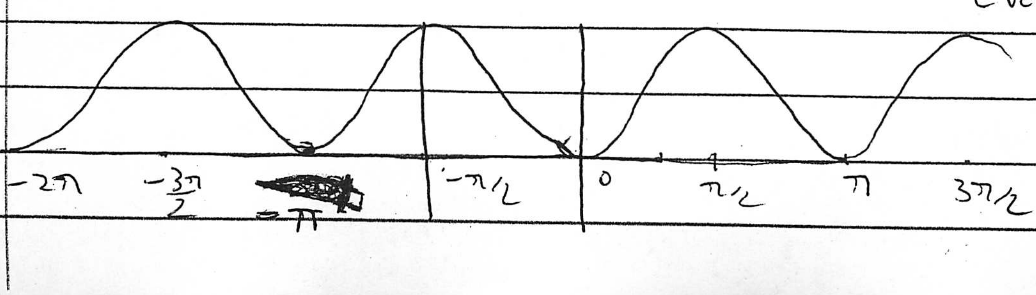
$$= \frac{2}{\pi} \int_0^{\pi} \left[\frac{\sin[(n+1)x] - \sin[(n-1)x]}{2} \right] dx$$

$$= \frac{2}{\pi} \left[\frac{\cos \pi n \cos \pi - 1}{n^2 - 1} \right]$$

$$a_n = \frac{-4}{\pi(n^2-1)} \quad \text{if } n \text{ is even}$$

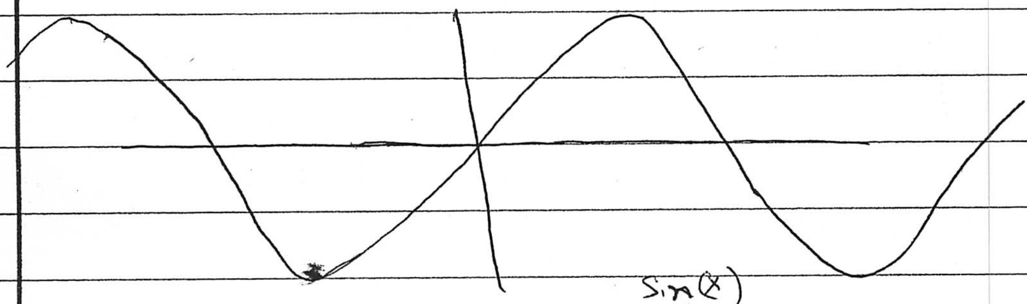
$$= 0 \quad \text{if } n \text{ is odd}$$

$$f(x) \sim \frac{2}{\pi}\left[1 - \sum_{n=1}^{\infty} \frac{2}{n^2-1} \cos nx\right]$$ is the even extn.



odd extension is just

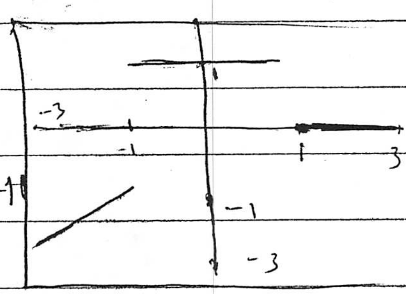
$f(x) \sim \sin x$



1.3

(2d)

$$f(x) = \begin{cases} 0 & 1 < x < 3 \\ 1 & -1 < x < 1 \\ x & -3 < x < -1 \end{cases}$$



The value take by the series

at $x = 3, -3$ $\quad \frac{1}{2}\{f(3-) + f(-3+)\} = \frac{-3}{2}$

at $x = 1$ $\quad \frac{1}{2}\{f(1+) + f(1-)\} = \frac{1}{2}$

at $x = -1$ $\quad \frac{1}{2}\{f(-1+) + f(-1-)\} = 0$

(8)

Suppose it coincides with $P(x)$

We know that $\sin(n\pi) = 0$

So $\sum \frac{1}{n^3} \sin(nx) = 0$ at $x = 0, \pi, 2\pi$

$\therefore$ P has zeros $0, \pi, 2\pi$

$\therefore$ $P(x) = a\, x(x-\pi)(x-2\pi)$

We also have

$\frac{1}{\pi} \int_0^{2\pi} P(x) \sin(nx)\, dx = \frac{1}{n^3}$

$\therefore$ L.H.S. $= 12k/n^3$ (after calculating the integral)

$\therefore$ $k = 1/2$

$\therefore$ $P(x) = \frac{1}{12} x(x-\pi)(x-2\pi)$