MAT 307 FALL 2012 FINAL EXAM

Name: **ID Number:**

Problem	1	2	3	4	5	6	7	8	9	Total
Score										

Directions: Some problems are easier, some harder. Do all of your work on these pages and cross out any work that should be ignored. Use the backs of pages if needed, indicating where you've done this. Notes, books, calculators and discussions with others are not permitted.

1. (15pts) Evaluate the integral

$$\int_D y dA,$$

where D is the domain $0 \leq x \leq 1$, $y \geq x^2$ and $y \leq x$ in $\mathbb{R}^2$.

2. (20pts) Consider the vector field $F = (ze^{xz}, 1, xe^{xz})$ on $\mathbb{R}^3$.

a. Find a potential function for F.

b. Using any method, evaluate the integral of F along the helix curve $\gamma(t) = (\cos t, \sin t, t)$, for $0 \leq t \leq \pi$.

c. Using any method, evaluate the integral of F along the straight line segment from $(1, 0, 0)$ to $(-1, 0, \pi)$.

3. (20pts) Find the volume of the region bounded by or between the paraboloids $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$.

4. (20pts) **(a).** State Stokes' Theorem for surface integrals in $\mathbb{R}^3$.

(b). Verify Stokes' theorem is true for the surface S equal to the part of the paraboloid $z = x^2 + y^2$ with $0 \leq z \leq 4$ and vector field $V = (-y, x, z^2)$.

5. (20pts) Suppose R is a smooth bounded region in $\mathbb{R}^3$ with boundary ∂R a smooth closed surface S in $\mathbb{R}^3$. Next suppose F is a smooth vector field on R and at any point $\underline{x}$ of the boundary $\partial R = S$, the vector $F(\underline{x})$ is tangent to the boundary surface S.

a. Compute the integral

$$\int_R divF dV.$$

Explain your answer.

b. Use your answer in (a) to compute

$$\int_R divF dV,$$

where R is the unit ball $x^2 + y^2 + z^2 \leq 1$ and $F(x, y, z) = (x^4 + y^3 e^z)(-y, x, 0)$.

6. (15pts) Compute the integral

$$\int_\gamma V,$$

where $V(x, y) = (0, x)$ and γ is the boundary of the rectangle $-1 \leq x \leq 2$, $-1 \leq y \leq 4$ traversed counterclockwise. It may be faster to compute this integral using Green's theorem.

7. (20pts) An $n \times n$ matrix A is called nilpotent if

$$A^k = 0$$

for some positive integer $k > 1$.

a. Show that the matrix

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

is nilpotent.

b. If A is nilpotent, explain why

$$detA = 0.$$

c. If A is nilpotent, can A be an invertible matrix? Explain why or why not.

8. (15pts) Find the tangent plane to the surface

$$x^2 + y^2 + e^z = 3$$

at the point $(1, 1, 0)$.

9. (15pts) Sketch the level curves and gradient vector field of the function on $\mathbb{R}^2$ given by

$$f(x, y) = \frac{x^2}{y}.$$

Describe also the set of points where the function has a local maximum or minimum, if any.