MAT 362 SPRING 05 HOMEWORK 6

Due Thursday, March 30

1. Do Problem 6.3, p.126 in the text. You can use the information given in the Solutions part of the text, but make sure you understand this argument. Its important.

2. Do Problem 6.17, p.140 of the text. Again you can use the solution in the back, but make sure you do all the details and understand this argument also.

3. Suppose the surface S is given locally as the graph of a function $f(x, y)$:

$$S = \{(x, y, z) \in \mathbb{R}^3 : z = f(x, y)\}.$$

Suppose $f(0, 0) = 0$ and $f_x(0, 0) = 0$, $f_y(0, 0) = 0$.

(a). Show that the origin is in S, and the tangent plane to S at the origin is the (x, y)-plane.

(b). Find the first fundamental form of S in the local coordinates (x, y).

(c). Find the second fundamental form of S at the origin.

4. The surface described by

$$\sigma(u, v) = (u - \frac{u^3}{3} + uv^2, v - \frac{v^3}{3} + vu^2, u^2 - v^2)$$

is called Enneper's surface.

(a). Find the second fundamental form of Enneper's surface in this chart.

(b). Find the principal curvatures.

(c). Show that the sum of the principal curvatures (called the mean curvature) is 0 everywhere. This means that Enneper's surface is a minimal surface.

5. A minimal surface is a surface for which $\kappa_1 + \kappa_2 = 0$ everywhere, where κ_i are the principal curvatures. Suppose S is a minimal surface for which the second fundamental form never vanishes. Prove that S has no umbilic points.