MAT 362 SPRING 05 HOMEWORK 4

Due Thursday, March 2

1. Read Sections 4.4 through 4.6 of the text - mainly examples. Let me know if you have any questions.

2. Let the surface S be given by the equation $f(x, y, z) = 0$, where 0 is a regular value of f. Show that the equation of the tangent plane to S at (x_0, y_0, z_0) is given by

$$f_x(x_0, y_0, z_0)(x - x_0) + f_y(x_0, y_0, z_0)(y - y_0) + f_z(x_0, y_0, z_0)(z - z_0) = 0.$$

3. Show that the equation of the tangent plane of surface which is the graph of a C^1 function $z = f(x, y)$ at the point $p_0 = (x_0, y_0)$ is given by

$$z = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

4. Show that if p is a point on a regular surface, one can choose coordinate axes (x, y, z) of $\mathbb{R}^3$ so that a neighborhood of p in S is of the form $z = f(x, y)$ where $f(0, 0) = 0$, $f_x(0, 0) = 0$ and $f_y(0, 0) = 0$, with $p = (0, 0, 0)$. (This is equivalent to taking the tangent plane of S at p as the xy-plane).

5. (20pts) For $a, b, c \neq 0$, show that each of the equations

$$x^2 + y^2 + z^2 = ax,$$
$$x^2 + y^2 + z^2 = by,$$
$$x^2 + y^2 + z^2 = cz,$$

defines a regular surface and that they all intersect orthogonally.