MAT 362 SPRING 05 HOMEWORK 3

Due Thursday, Feb. 23

1. Let S_1 and S_2 be smooth surfaces. Prove that the "intrinsic" definition of a smooth map $F: S_1 \to S_2$ is independent of any choice of local charts.

2. Show that the paraboloid $z = z^2 + y^2$ is diffeomorphic to the plane.

3. Let $f(x, y, z) = z^2$. Prove that 0 is not a regular value and yet $f^{-1}(0)$ is a regular surface.

4. Consider the set of points S in $\mathbb{R}^3$ given by $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.
(a). Prove that S is a smooth surface.
(b). Show that

$$\sigma(u, v) = (au \cos v, bu \sin v, u^2), \ u > 0$$

is a local chart for S, omitting only one point.
(c). Set $a = 1$, $b = 2$ and sketch the surface. Sketch on the surfaces the lines where $u = const$ and $v = const$, i.e. the local coordinate curves on S induced by σ.

5. Show that the antipodal map $A(x, y, z) = (-x, -y, -z)$ is a diffeomorphism of the sphere $S^2(1)$ into itself. You can use either the intrinsic or extrinsic definition of smooth maps. How is A related to A^{-1}?