MAT 362 SPRING 05 HOMEWORK 2

Due Thursday, Feb. 16

1. Show that the cylinder $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1\}$ is a surface and find local charts forming an atlas for it. Describe the transition maps for the atlas.

2. Is the set $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1, \text{ and } z \geq 0\}$ a surface? Why or why not? Same question for the set $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1, \text{ and } z > 0\}$.

3. Let $P = \{(x, y, z) \in \mathbb{R}^3 : x = y\}$ and let $F : U \to \mathbb{R}^3$, be given by

$$F(u, v) = (u + v, u + v, uv),$$

where $U = \{(u, v) \in \mathbb{R}^2 : u > v\}$. Prove or disprove that F is a chart covering all of P.

4. Show that the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

is a surface in $\mathbb{R}^3$.

Hint: To this as with the sphere, for example with charts of the form

$$F(u, v) = (a \sin u \cos v, b \sin u \sin v, c \cos u).$$

5. Show that the sphere $S^2(1)$ given by

$$x^2 + y^2 + z^2 = 1$$

is homeomorphic (diffeomorphic) to the ellipsoid in Problem 4.